30% Chance of Rain?

Chaos and Numerical Weather Prediction

Outline

- Dynamical system
- State space
- Trajectory
- Attractor
- Lyapunov exponents
- Singular vectors

} What they are and why you should care

- State dependent error growth
- Implications of model inadequacy
- What multi-models buys us

} An ensemble-based approach

- A departure into the real world

Dynamical system

- A set of equations giving the time evolution (or ordered evolution) of the state of a system from knowledge of its previous history (in contrast to a stochastic system)

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$$

State space

- A dynamical system's prognostic variables expressed as a point in a multi-variate space.

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) \longrightarrow \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix}$$

Trajectory

- Given **F** and some **x**(0), one can determine subsequent **x**(t), or the trajectory of **x**(t), for all t by integrating the dynamical system.

Attractor

- The bounded region of state space towards which initial conditions eventually converge.

Example: Unforced pendulum

Dynamical system:

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + \omega_0^2 x = 0$$

Change of variable:

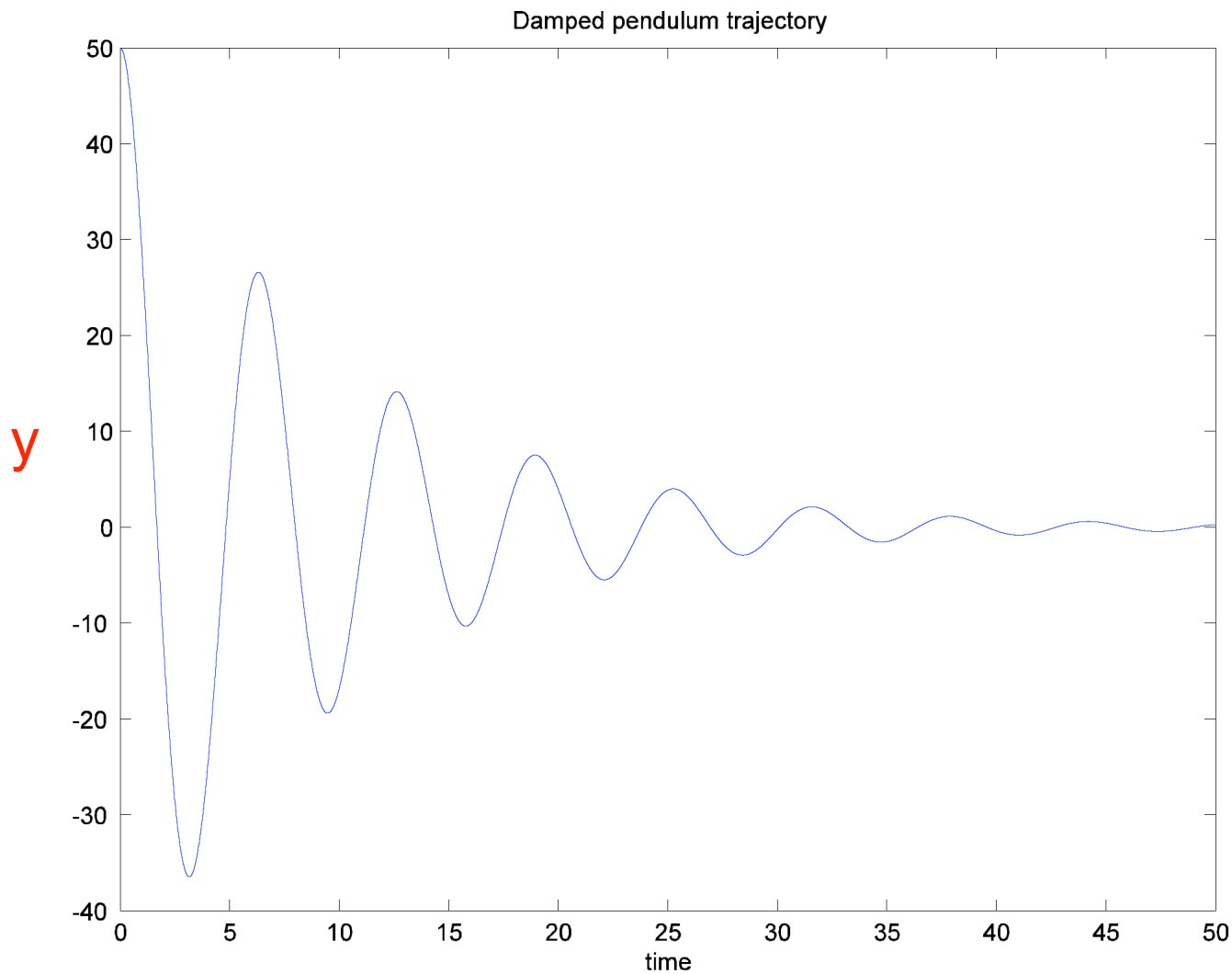
$$\left.\begin{aligned} y &= x \\ z &= \frac{dx}{dt} \end{aligned}\right\} \text{State space}$$

Two first-order ODEs:

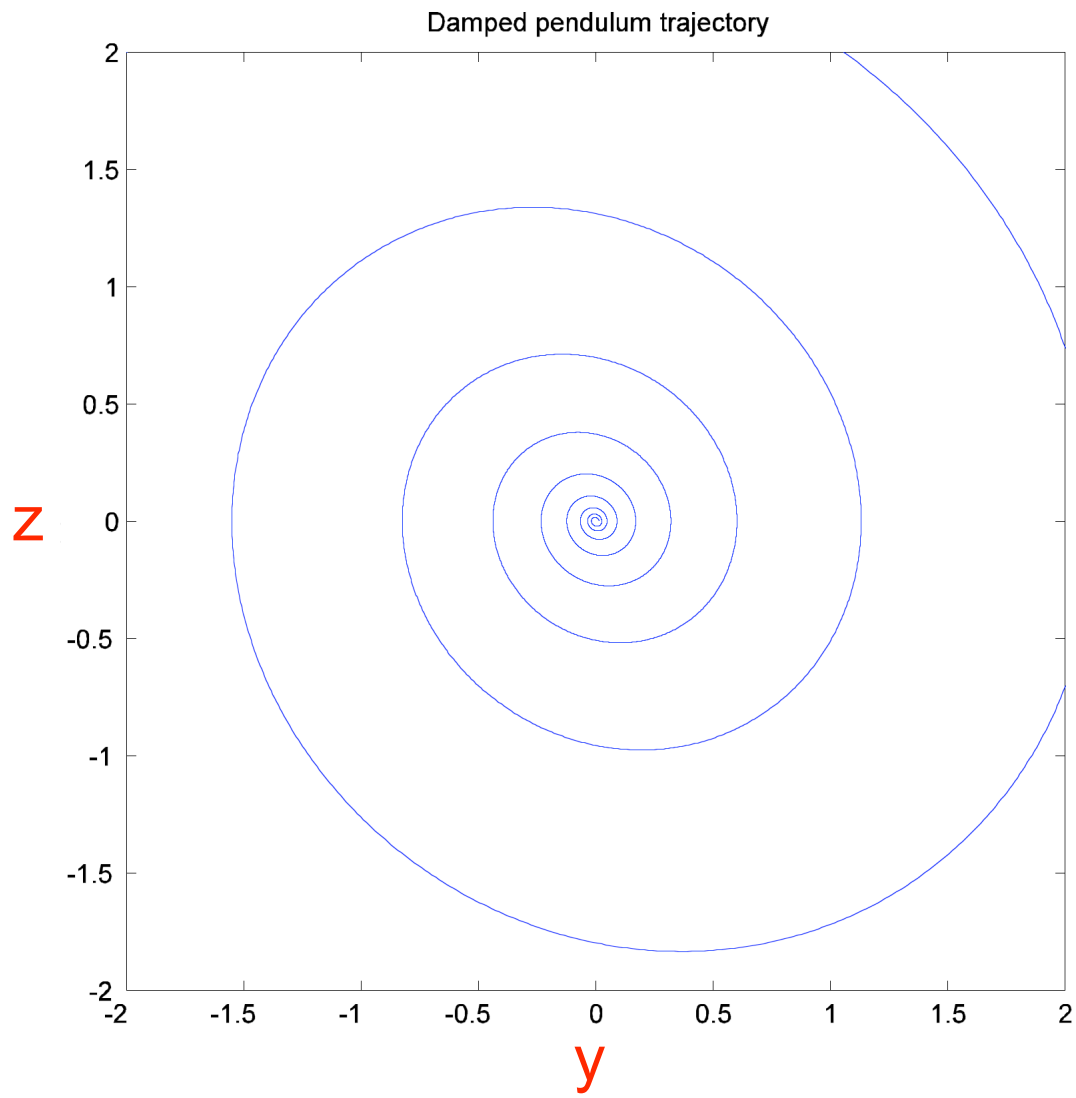
$$\frac{dy}{dt} = z$$

$$\frac{dz}{dt} = -bz - \omega_0^2 y$$

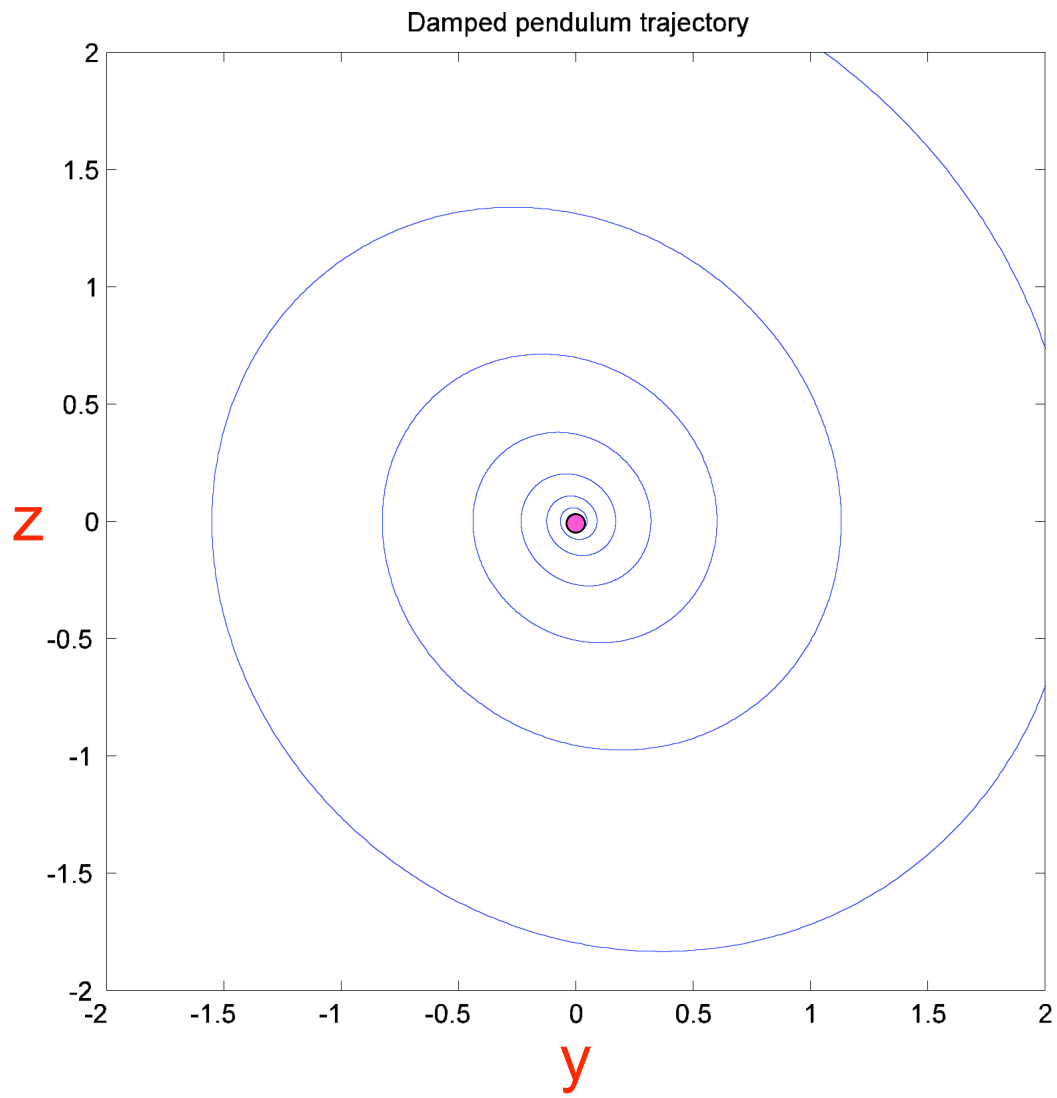
Trajectory: Unforced pendulum



Trajectory: Unforced pendulum



Attractor: Unforced pendulum



Example: Forced pendulum

Dynamical system:

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + \omega_0^2 x = f_0 \sin \omega t$$

Change of variable:

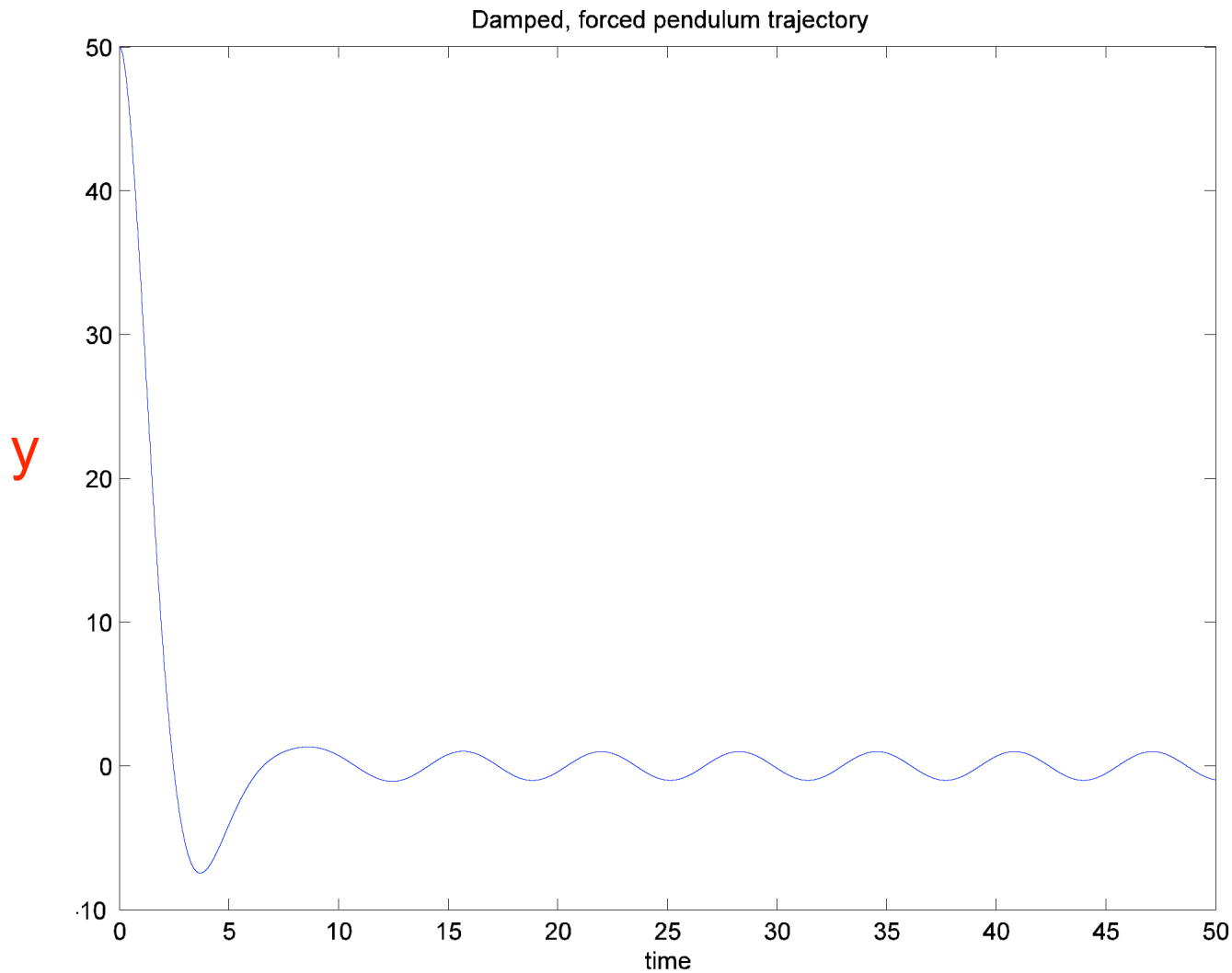
$$\left.\begin{aligned} y &= x \\ z &= \frac{dx}{dt} \end{aligned}\right\} \text{State space}$$

Two first-order ODEs:

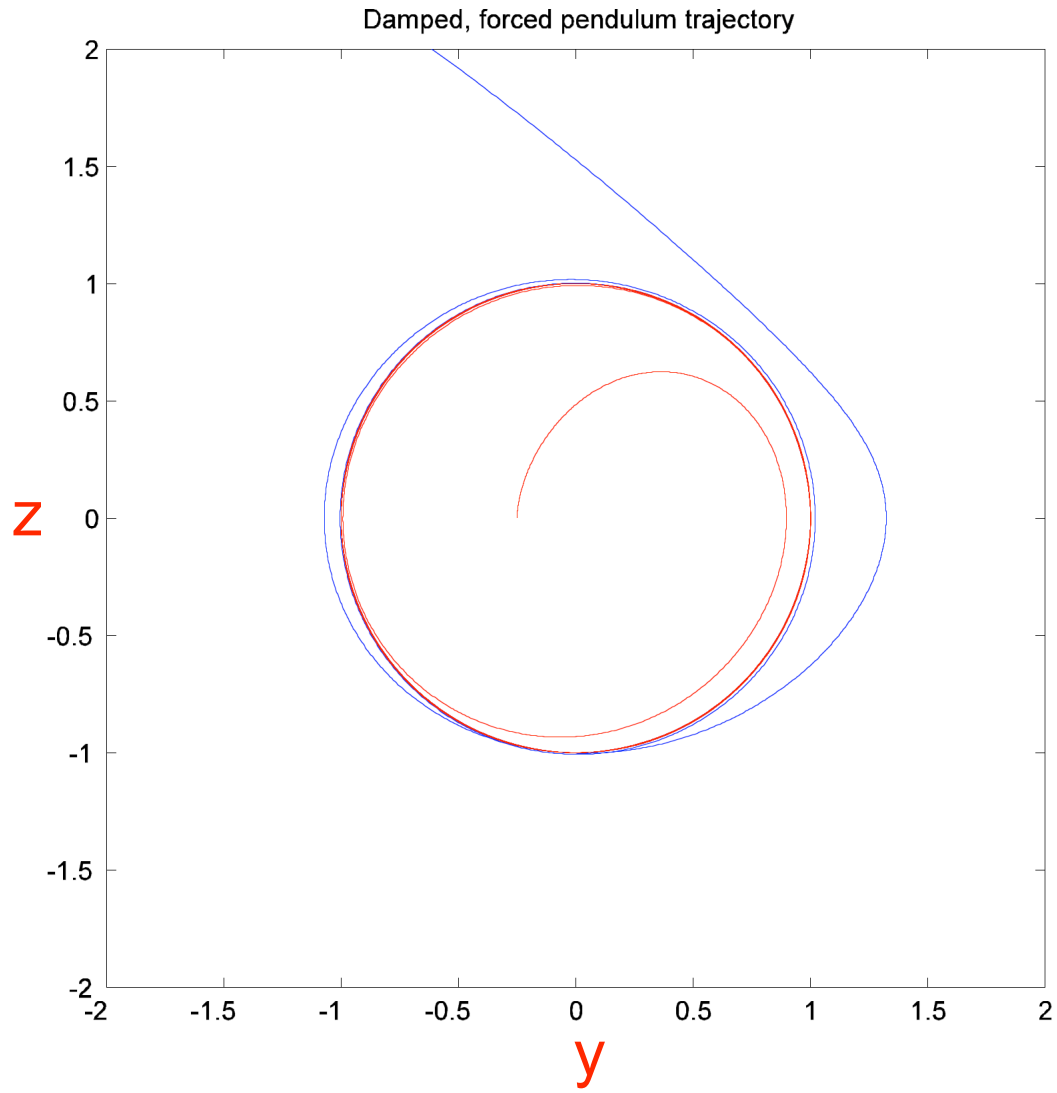
$$\frac{dy}{dt} = z$$

$$\frac{dz}{dt} = -bz - \omega_0^2 y + f_0 \sin \omega t$$

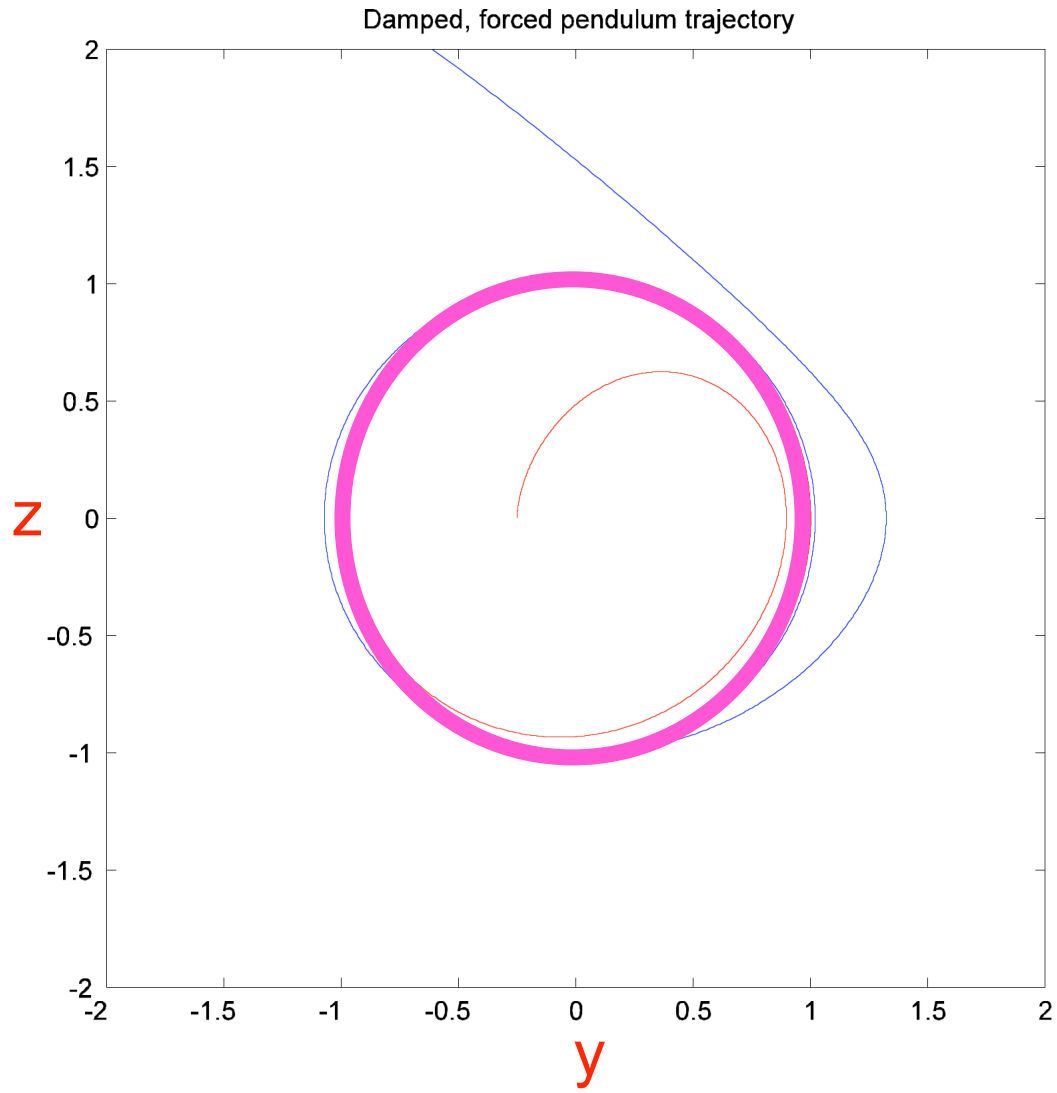
Trajectory: Forced pendulum



Trajectory: Forced pendulum



Attractor: Forced pendulum



Example: Lorenz '63

Dynamical system:

$$\frac{dx}{dt} = -\sigma x + \sigma y$$

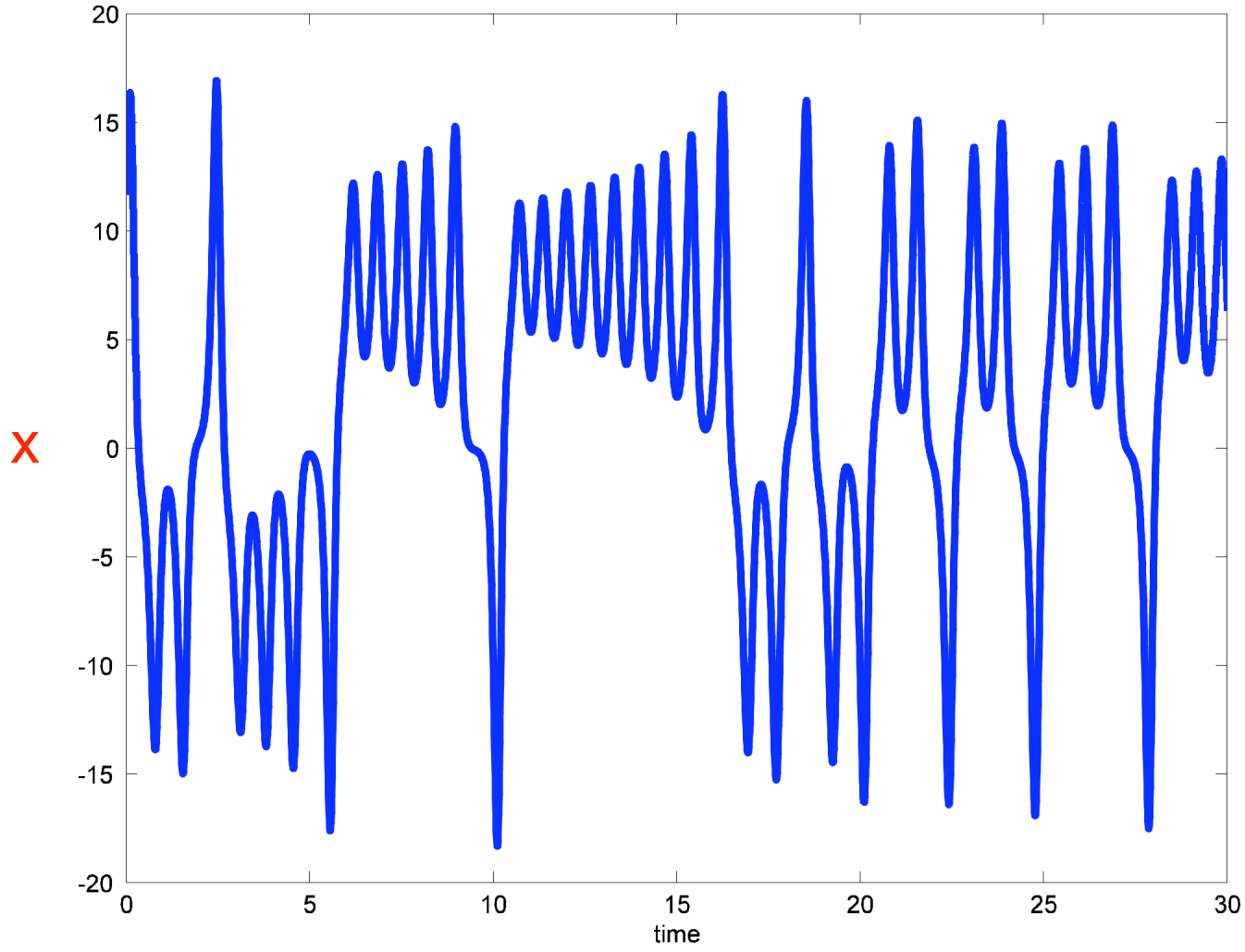
$$\frac{dy}{dt} = -xz + rx - y$$

$$\frac{dz}{dt} = xy - bz$$

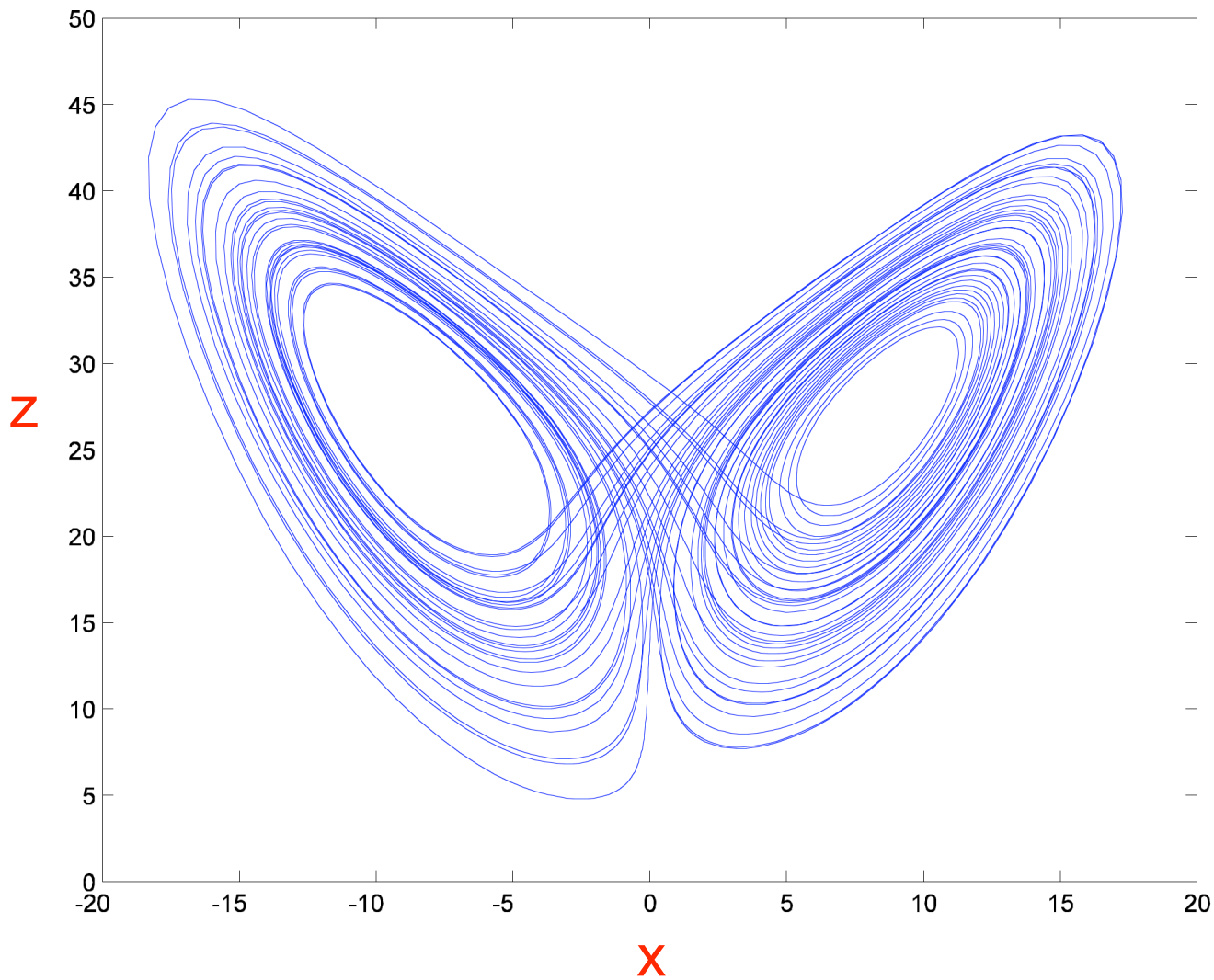
State space:

$$[x \; y \; z]^T$$

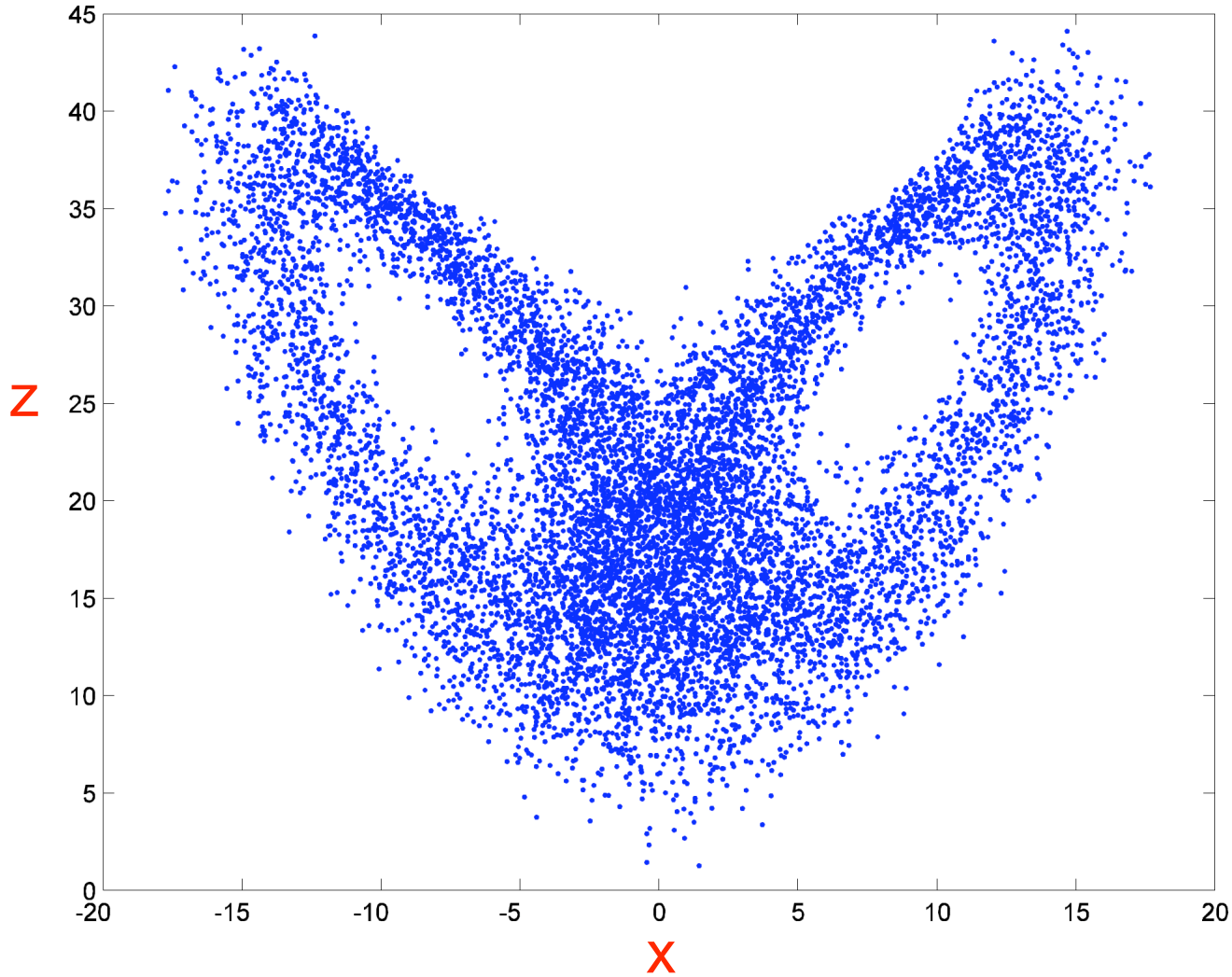
Trajectory: Lorenz '63



Trajectory: Lorenz '63



Attractor: Lorenz '63



Example: NWP

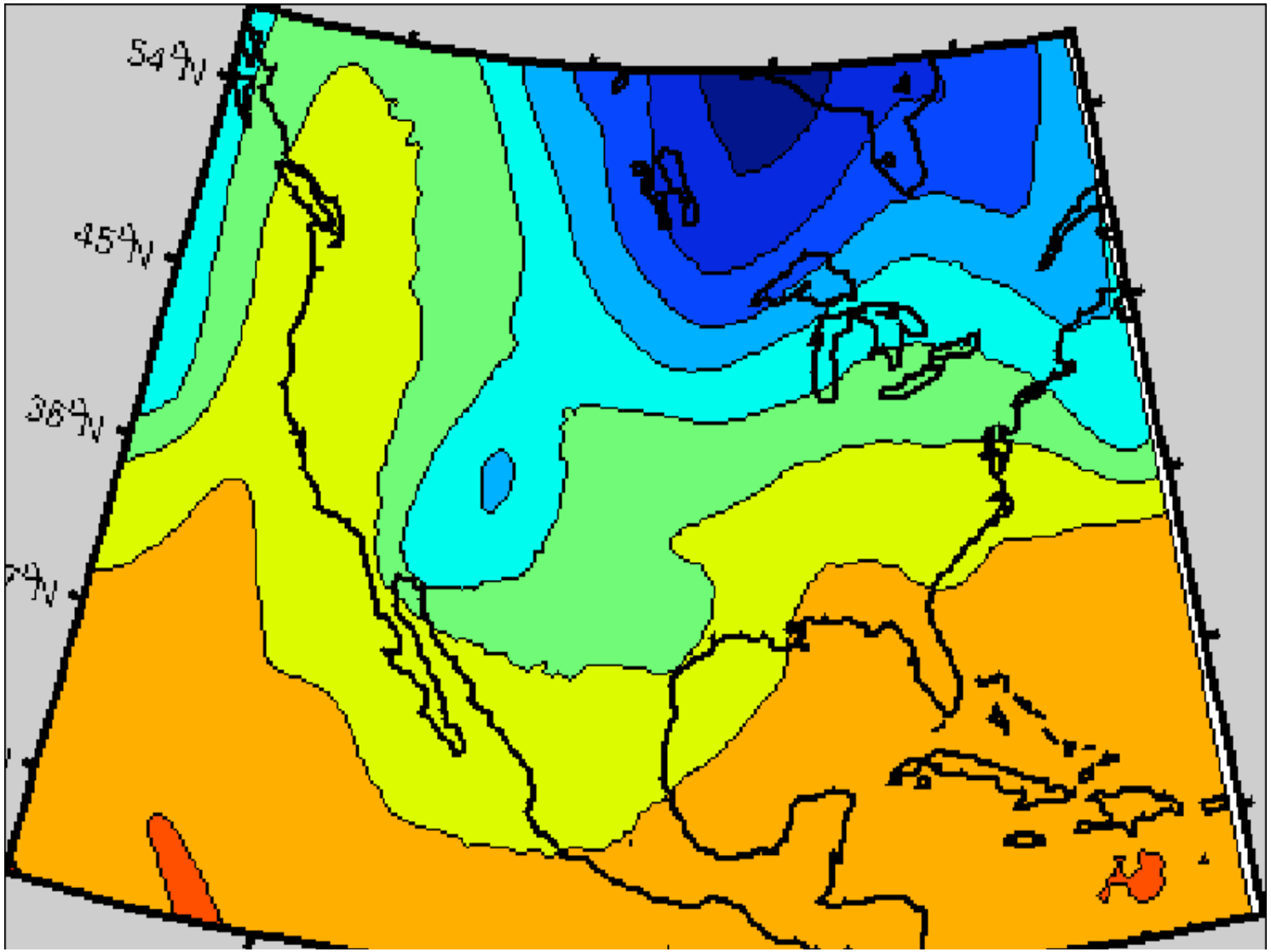
Dynamical system:

$$\frac{\partial u}{\partial t} = -\frac{u}{a\cos\varphi}\frac{\partial u}{\partial \lambda} - \frac{v}{a}\frac{\partial u}{\partial \varphi} - \left[\dot{\eta}\frac{\partial p}{\partial \eta}\right]\frac{\partial u}{\partial p} + uv\frac{\tan\varphi}{a}$$

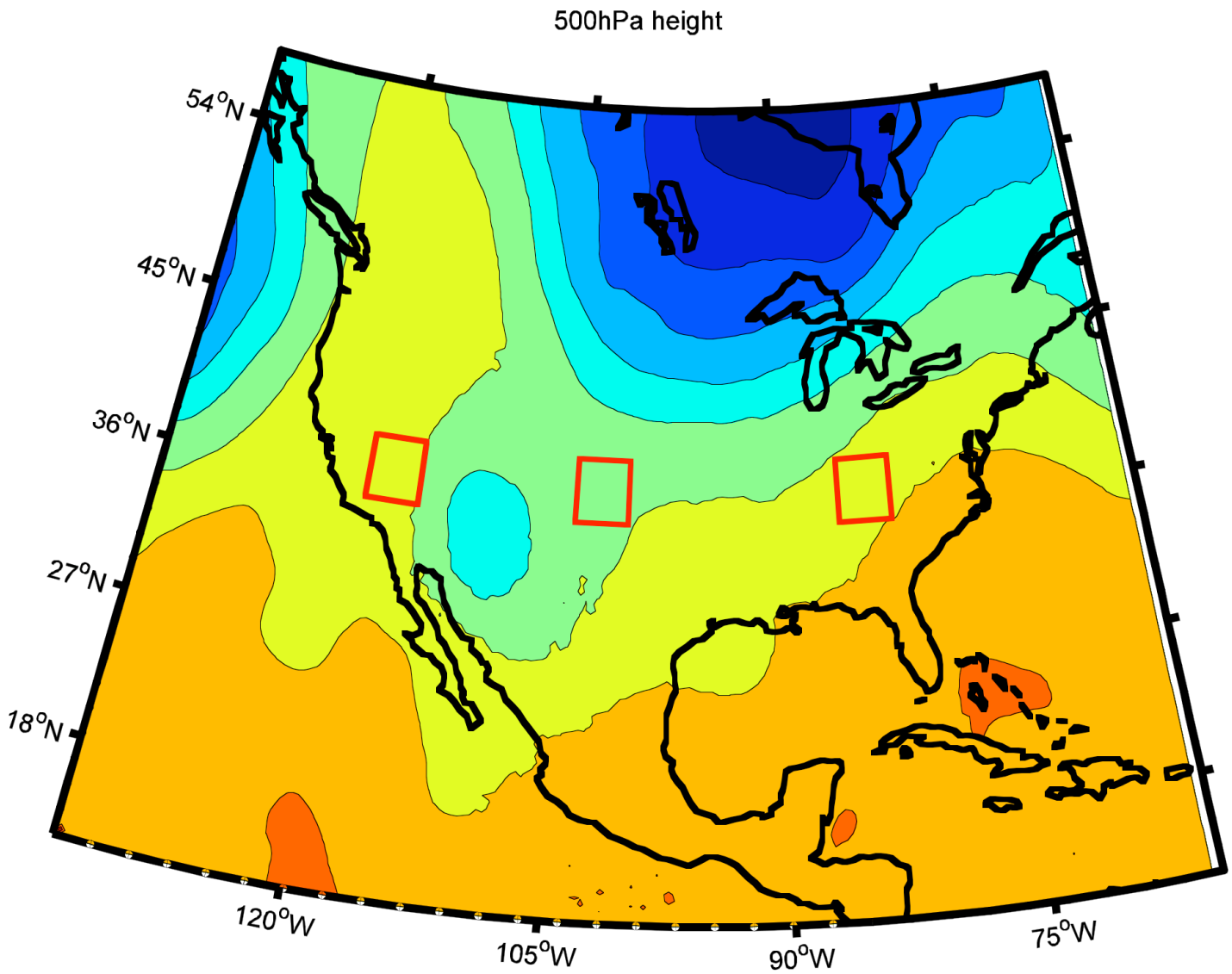
$$+ fv - \frac{1}{a\cos\varphi}\frac{\partial \varphi}{\partial \lambda} - \frac{1}{a\cos\varphi}\frac{\partial \varphi}{\partial \eta}\frac{\partial \eta}{\partial \lambda} + Q_u$$

State space: $[\tilde{\mathbf{n}}\ \mathbf{u}\ \mathbf{v}\ \mathbf{w}\ \mathbf{p}\ \mathbf{T}\ \mathbf{q}]^T$

Trajectory: NWP



Trajectory: NWP



Trajectory: NWP

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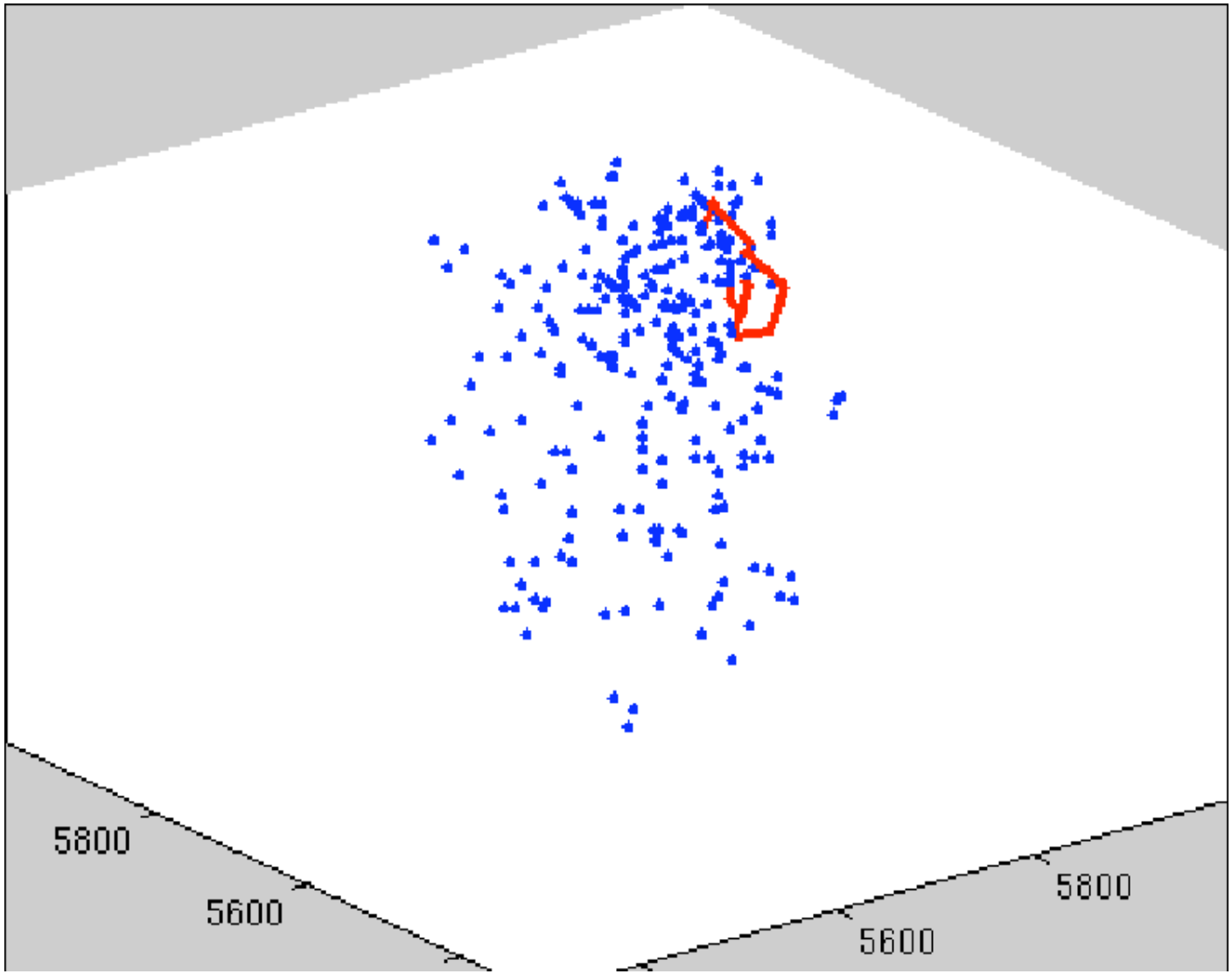


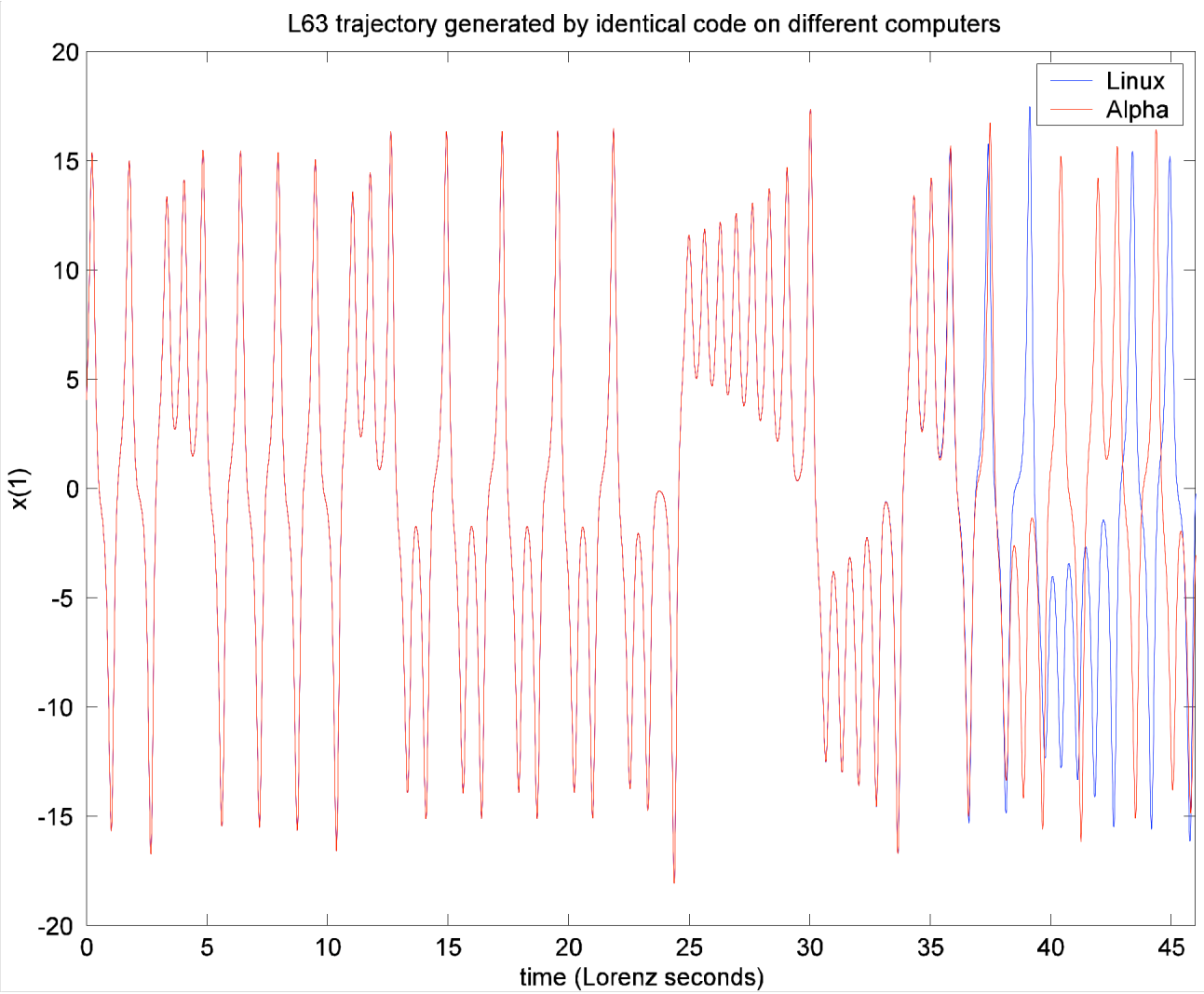
Lyapunov exponents

- A measure of the “amount” of chaos.
- Indicate the average rate of error growth for infinitesimal errors over an infinite time.

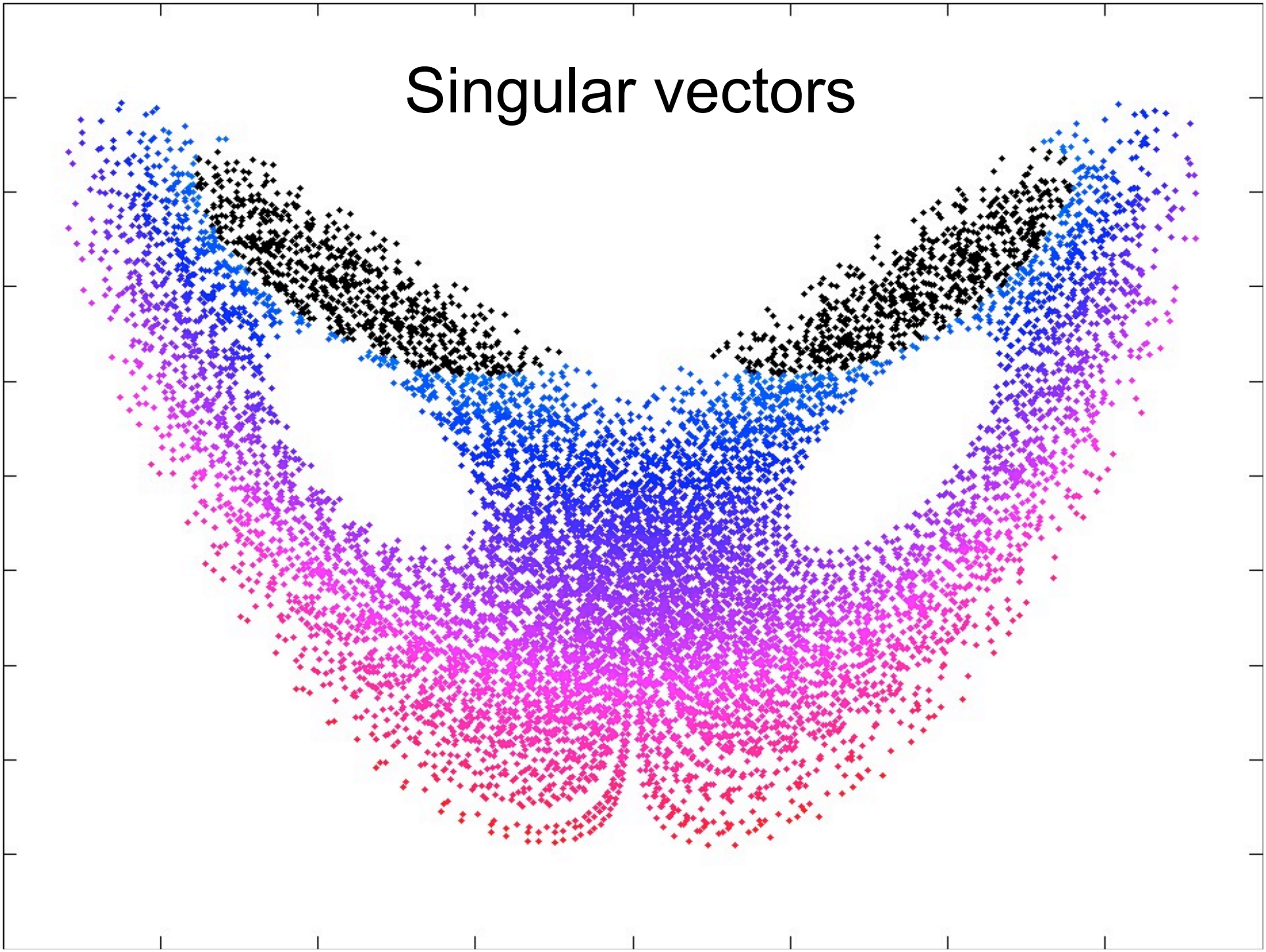
$$\mathring{\mathbf{a}}(t) = e^{\lambda t}\mathring{\mathbf{a}}(0)$$

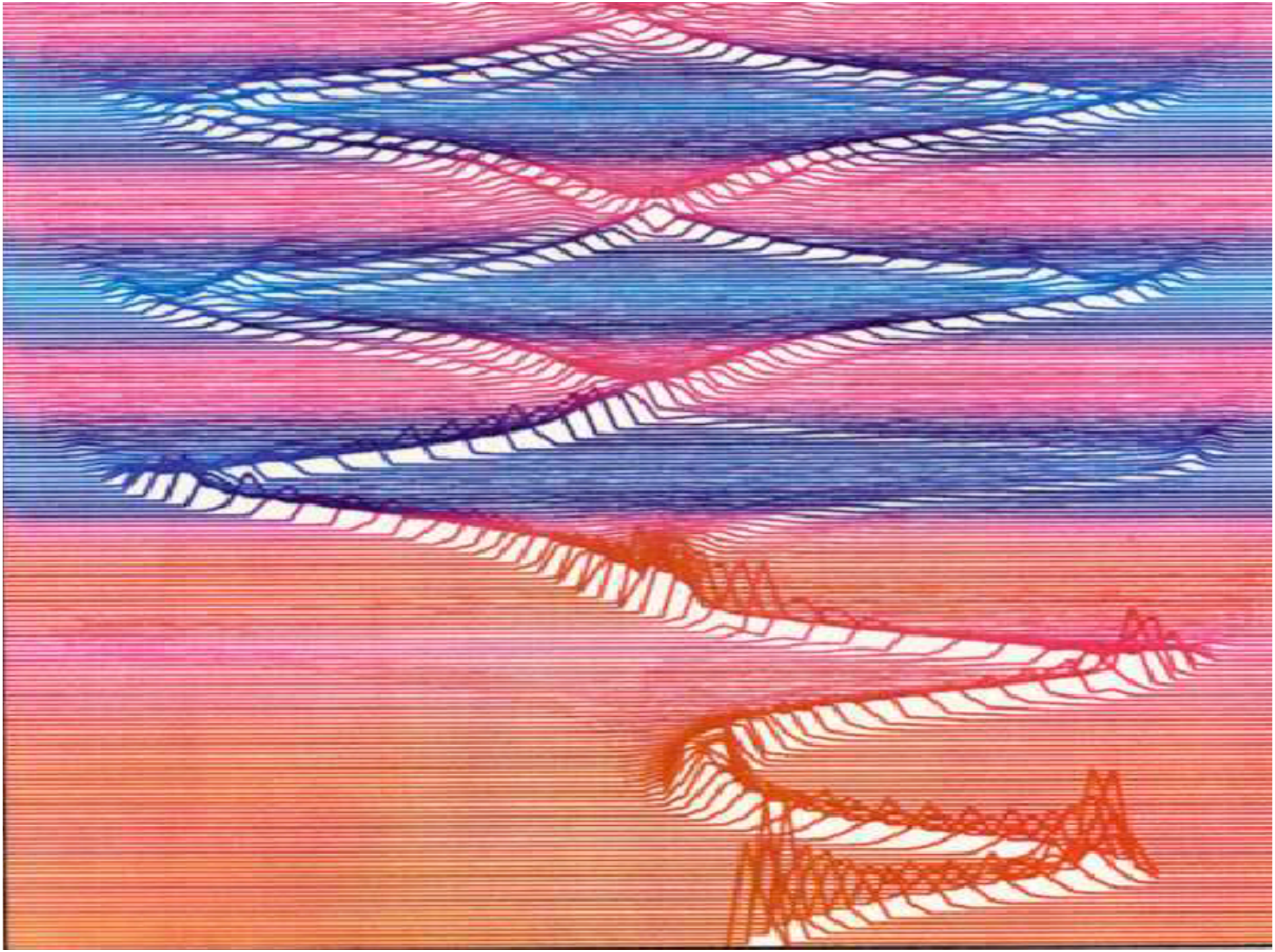
- Can be numerically estimated using linear theory.
- Lyapunov exponents are an invariant of the system; they are not state dependent (and need never be realized!)

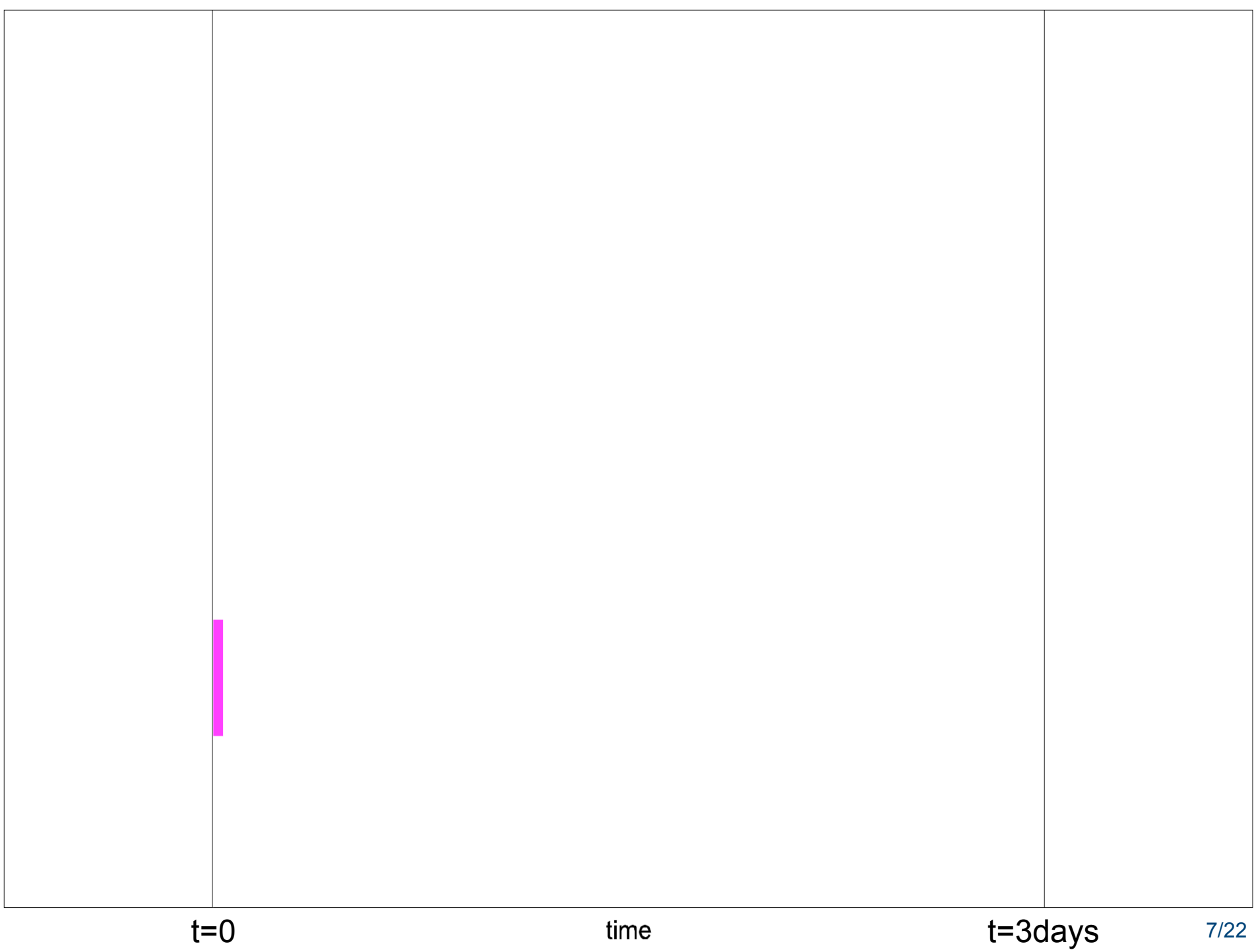
Chaos: SDTIC

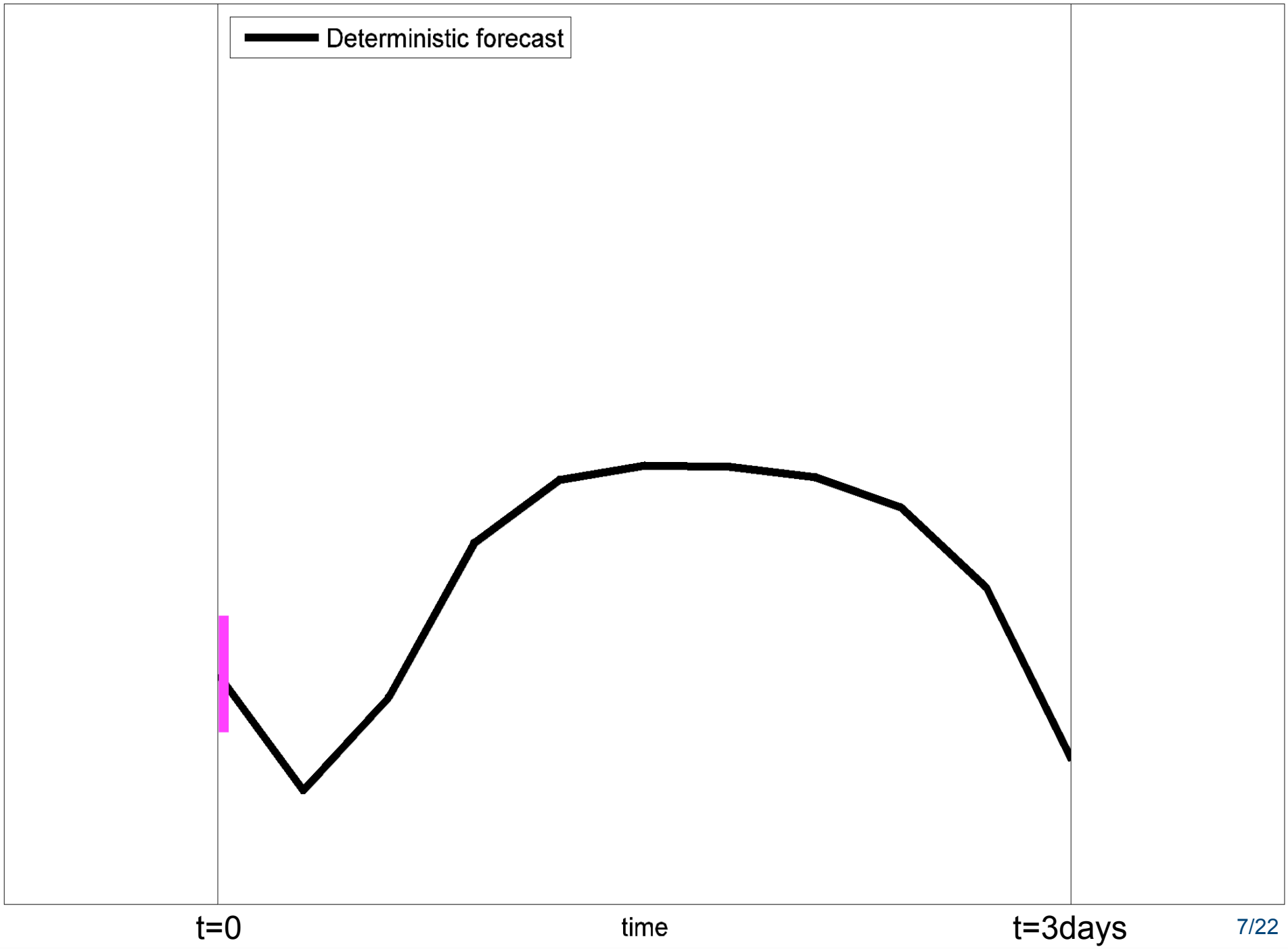

L63 trajectory generated by identical code on different computers
Linux
Alpha
x(1)
time (Lorenz seconds)
20
15
10
5
0
-5
-10
-15
-20
0
5
10
15
20
25
30
35
40
45

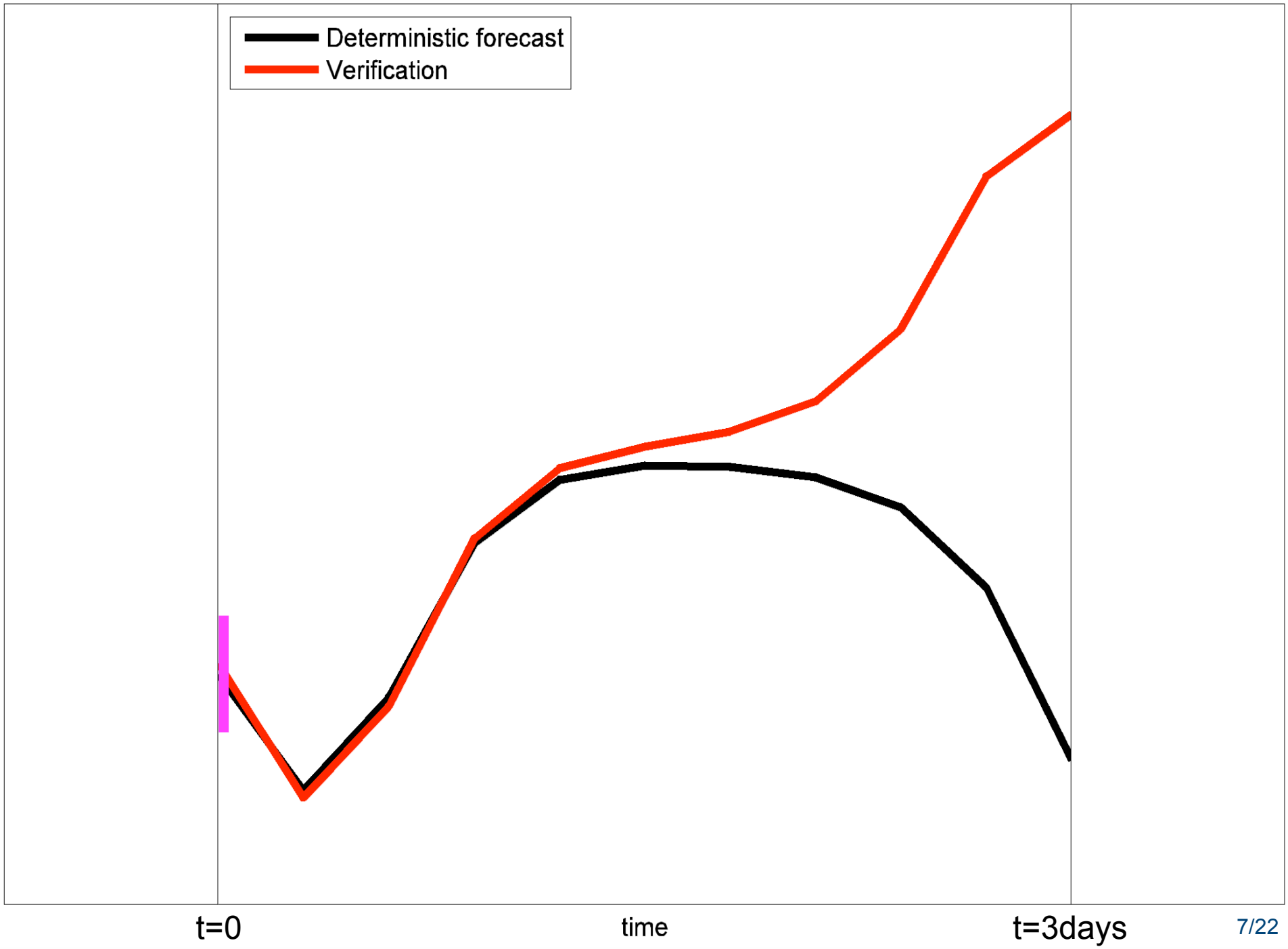
Singular vectors

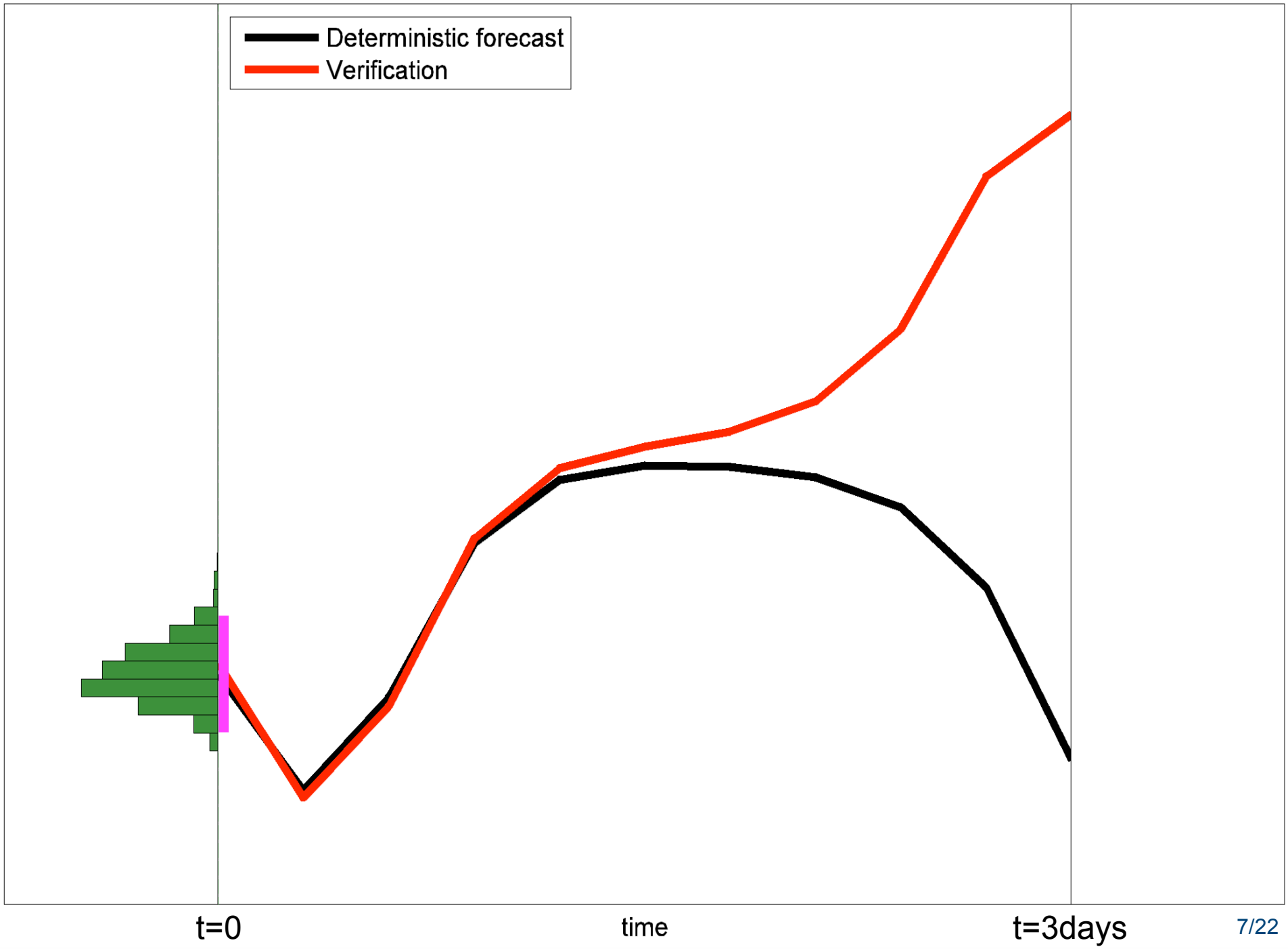


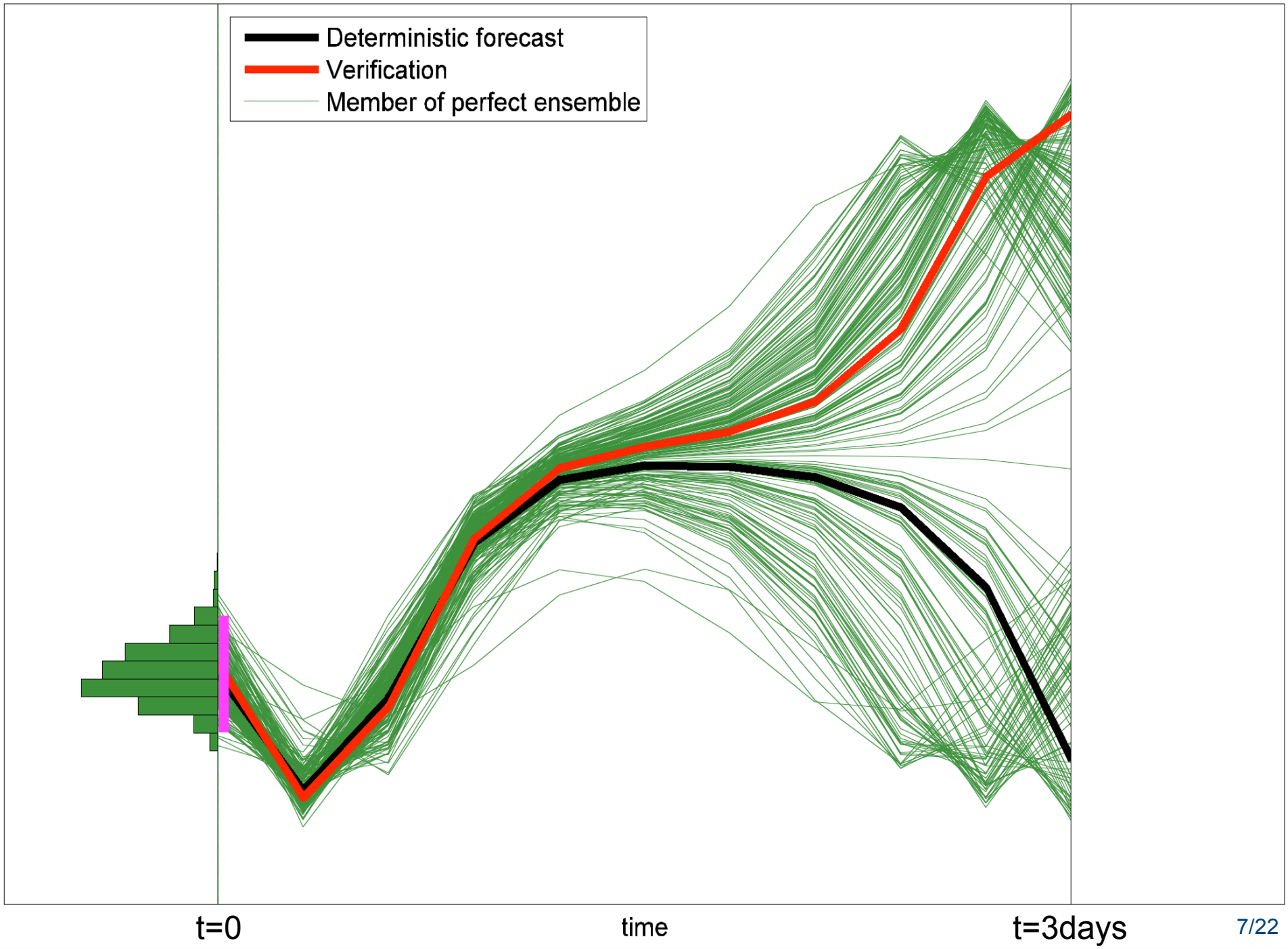


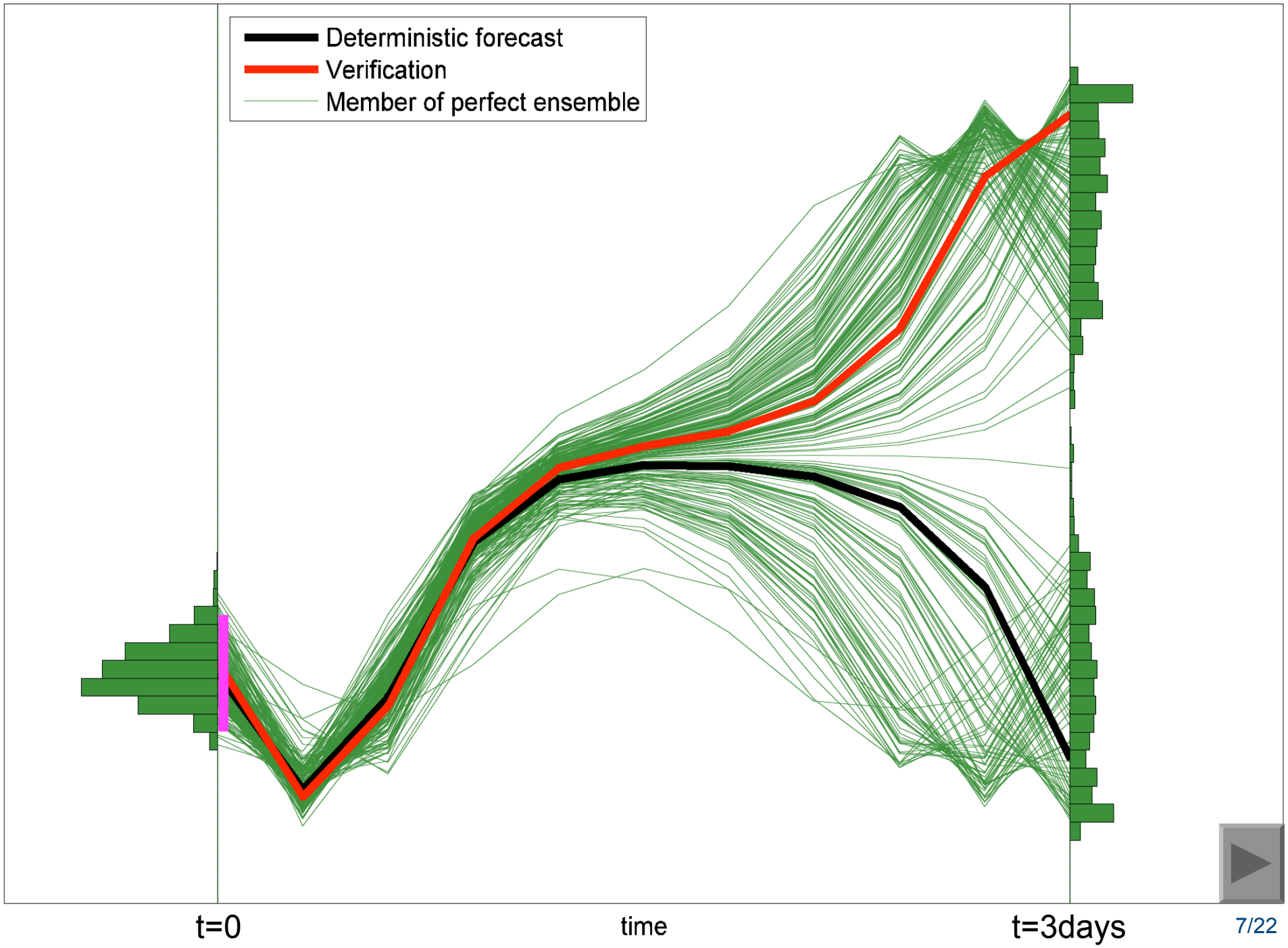

t=0
time
t=3days


Deterministic forecast
t=0
time
t=3days


Deterministic forecast
Verification
t=0
time
t=3days


Deterministic forecast
Verification
t=0
time
t=3days


Deterministic forecast
Verification
Member of perfect ensemble
t=0
time
t=3days

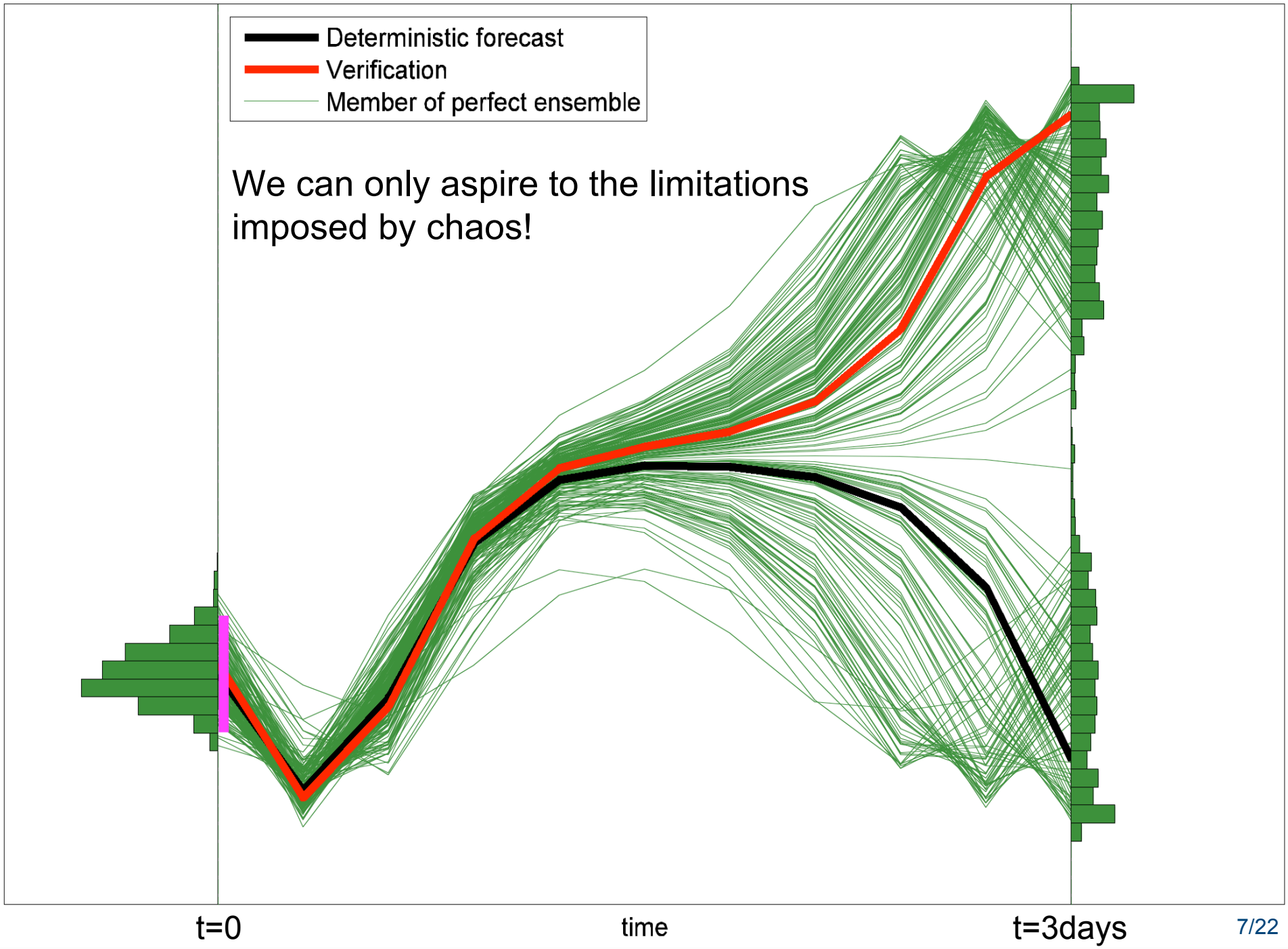

Deterministic forecast
Verification
Member of perfect ensemble
t=0
time
t=3days

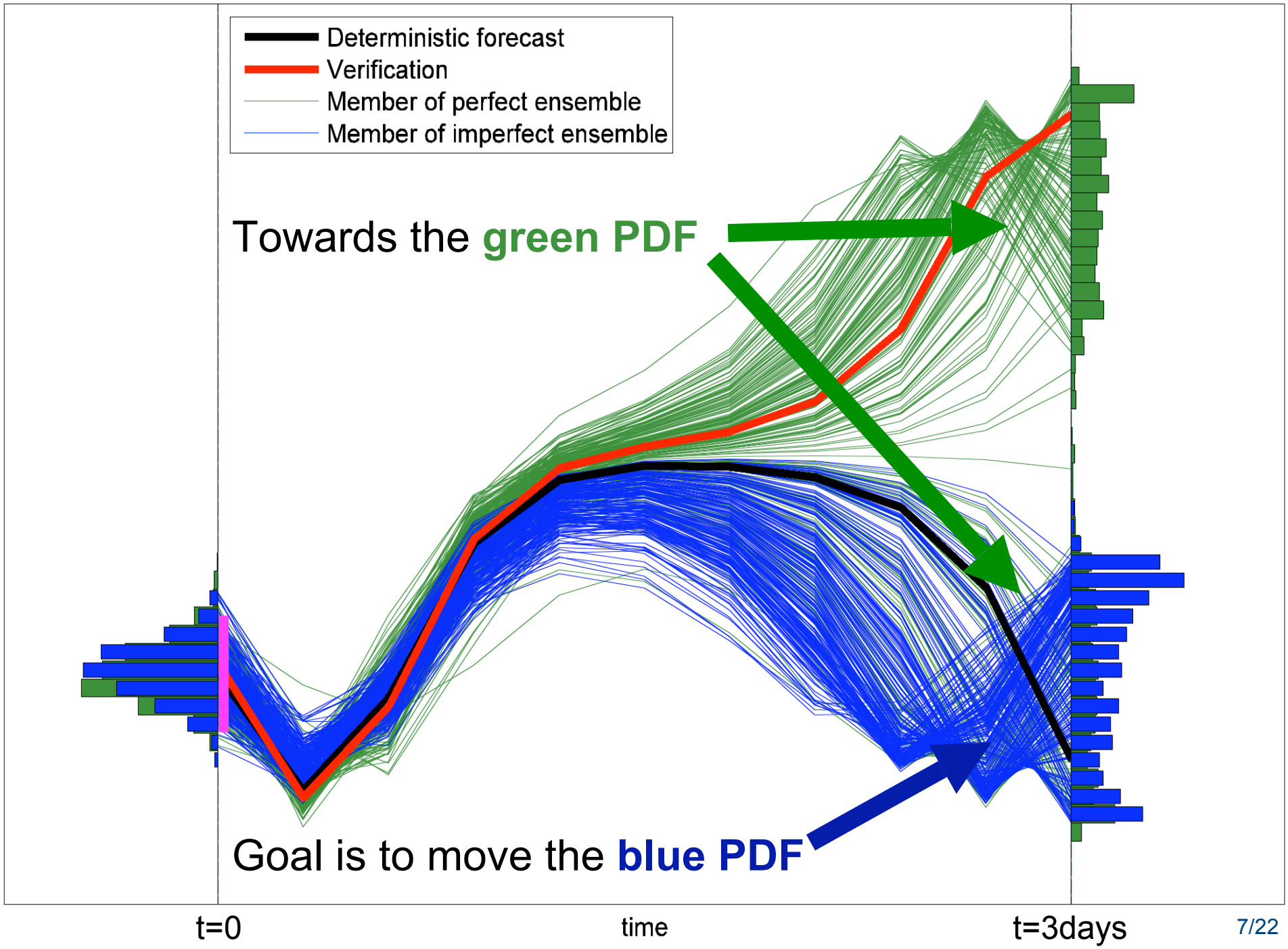
Now you have to make a choice.

You take the blue pill and the lecture ends, you wake up in your bed and happily do mathematics…

…or you take the red pill, and try to use ensemble guidance knowing all models are wrong.

"Remember that all I am offering is the truth. Nothing more" - Morpheus

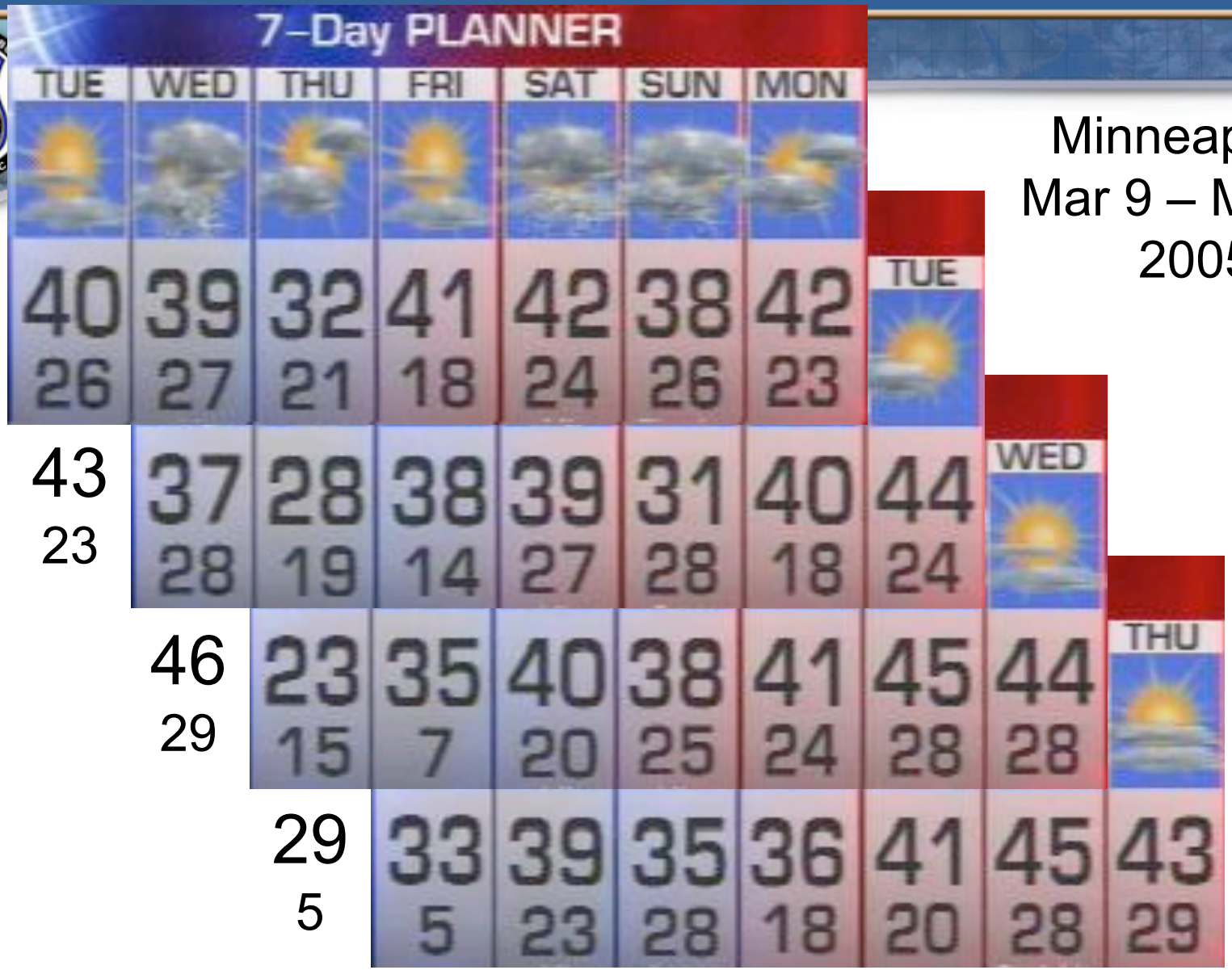

Deterministic forecast
Verification
Member of perfect ensemble
We can only aspire to the limitations imposed by chaos!
t=0
time
t=3days


Deterministic forecast
Verification
Member of perfect ensemble
Member of imperfect ensemble
Towards the green PDF
Goal is to move the blue PDF
t=0
time
t=3days

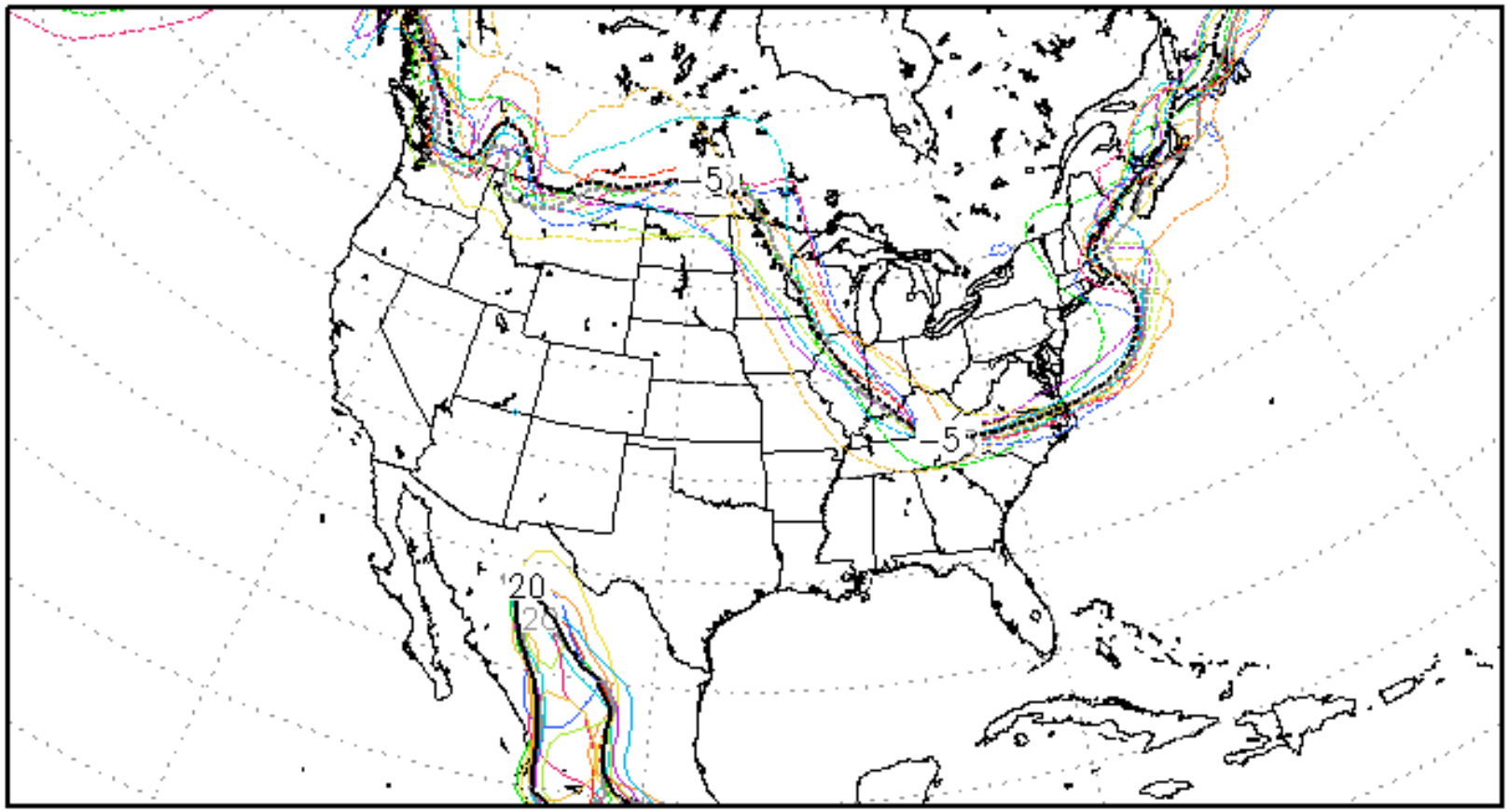
- 3d attractor pictures with 1 model
- Put an ensemble on there
- Multi-models
- Multi-models with ensembles

A brief foray into the real world

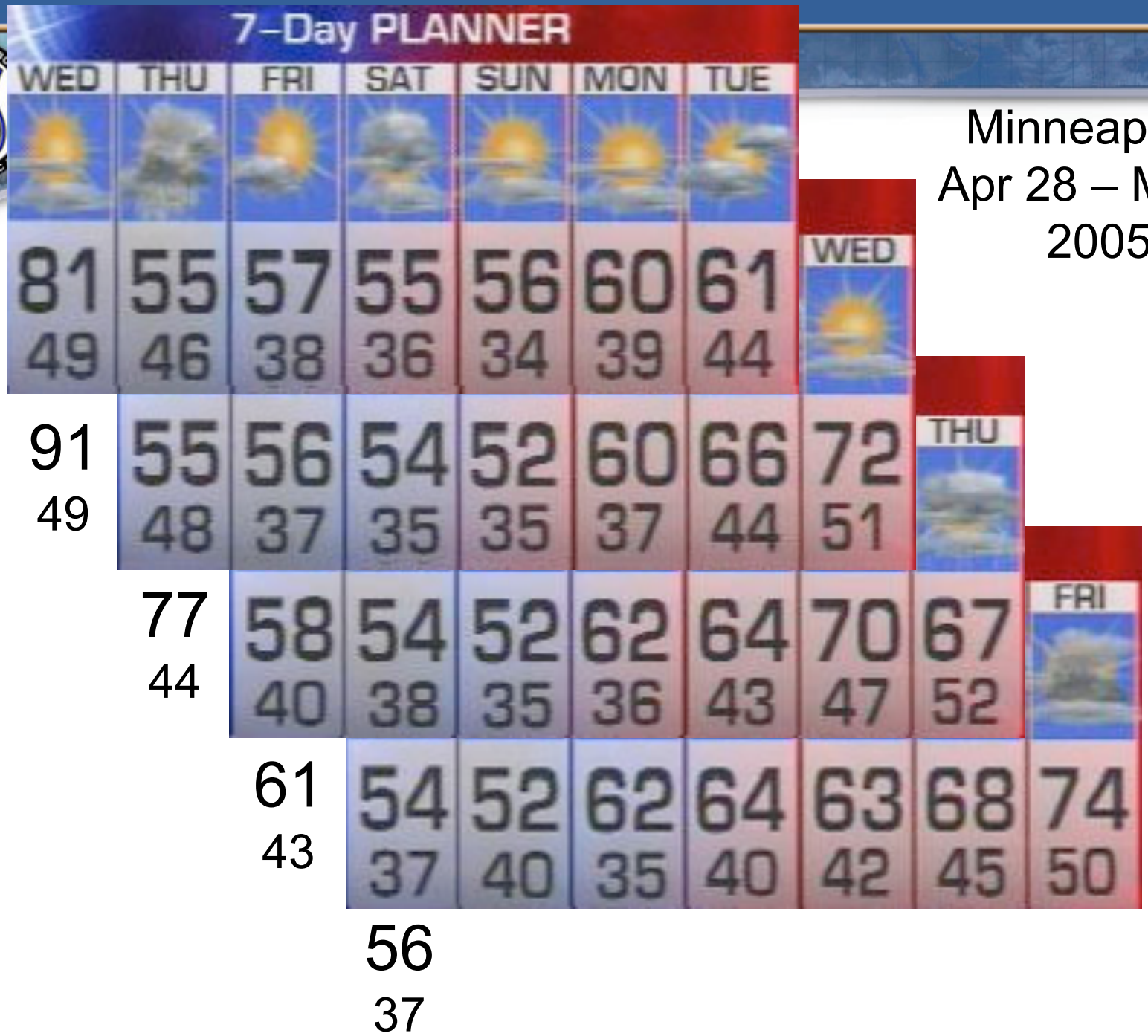
Minneapolis,
Mar 9 – Mar 15
2005

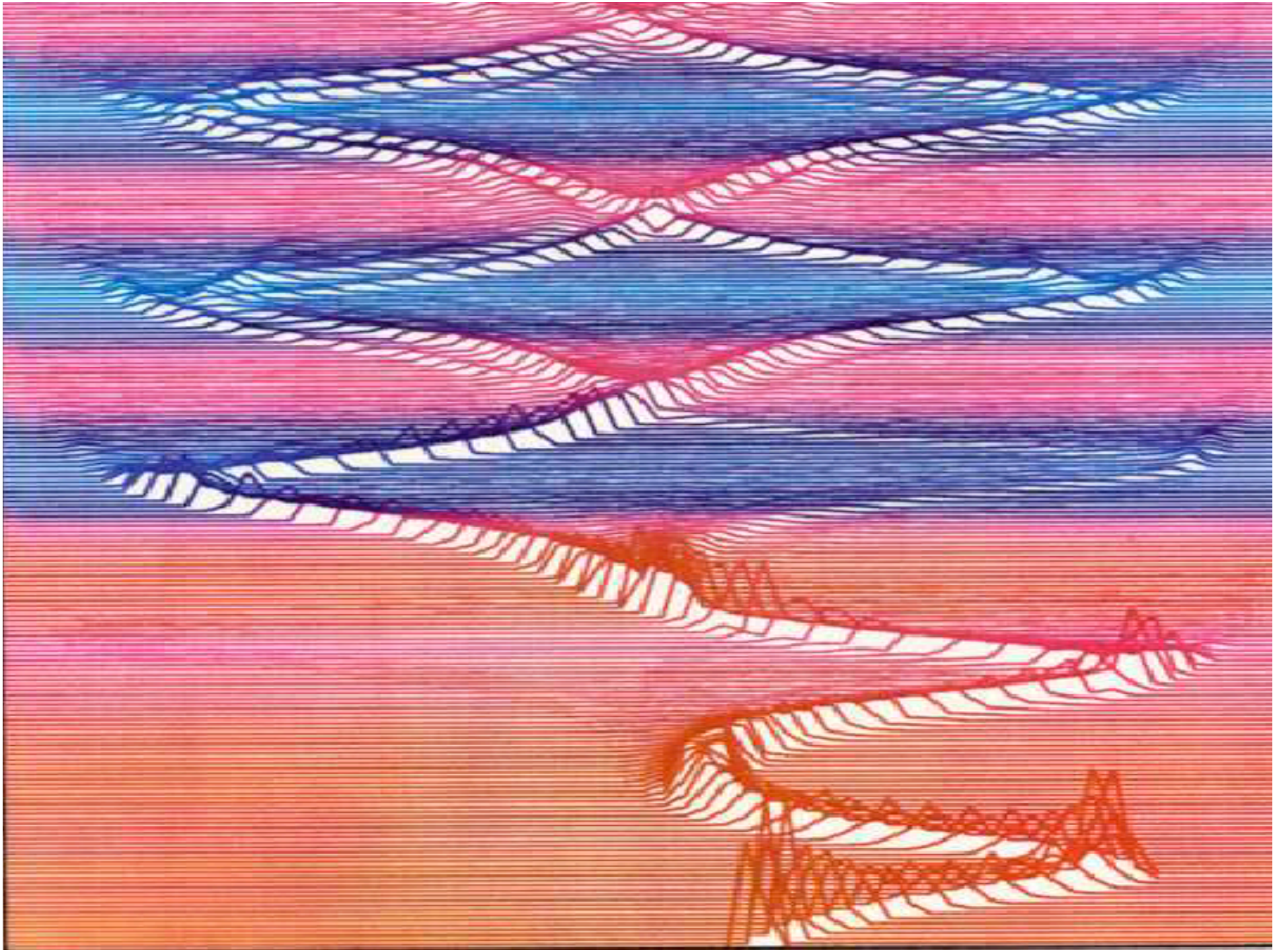


ens run for 00Z 10Mar2004 valid 00Z13MAR2004 −5 and 20 temp. contours at 850−hPa, 1x1 degree resolution

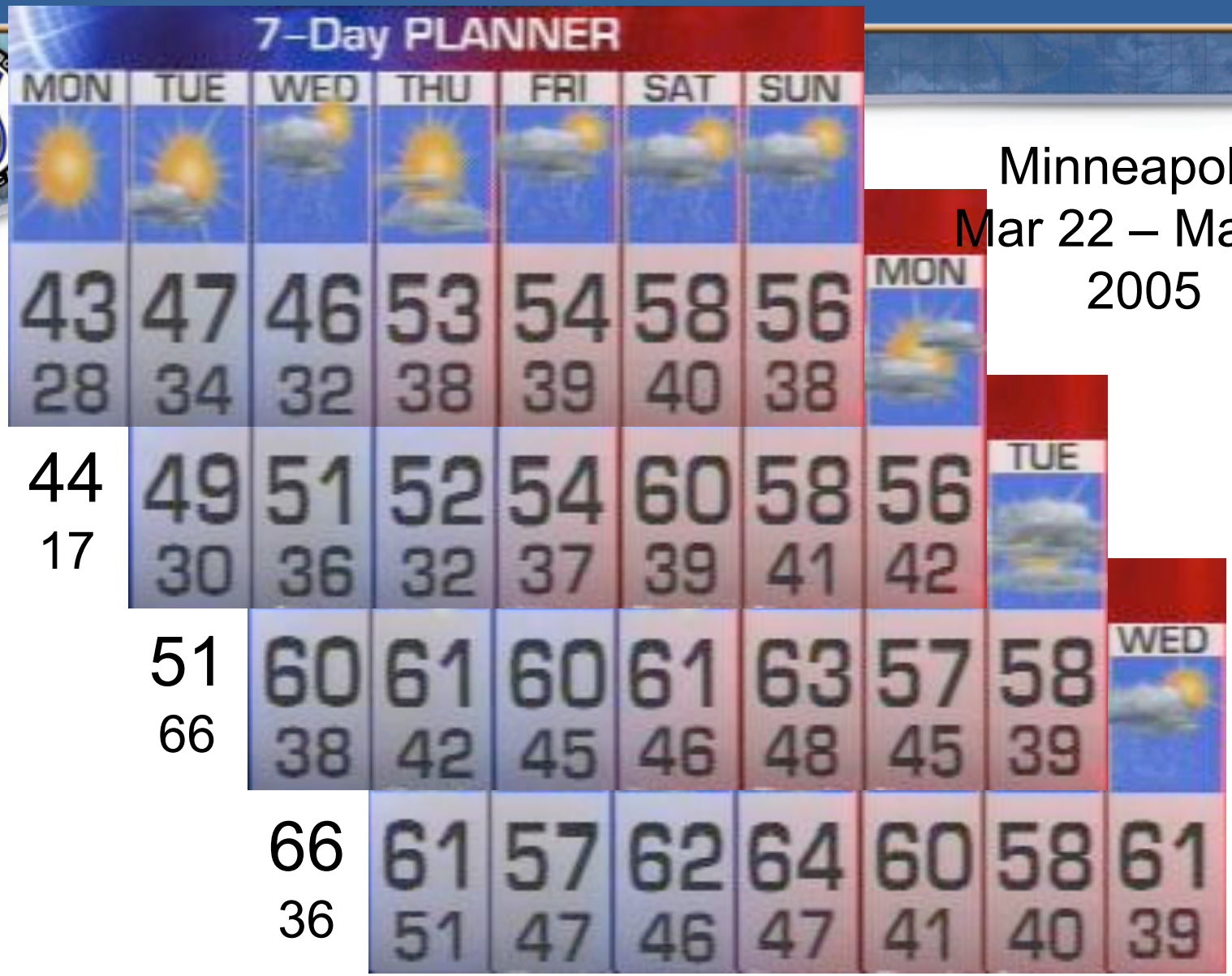


Minneapolis,
Apr 28 – May 4
2005



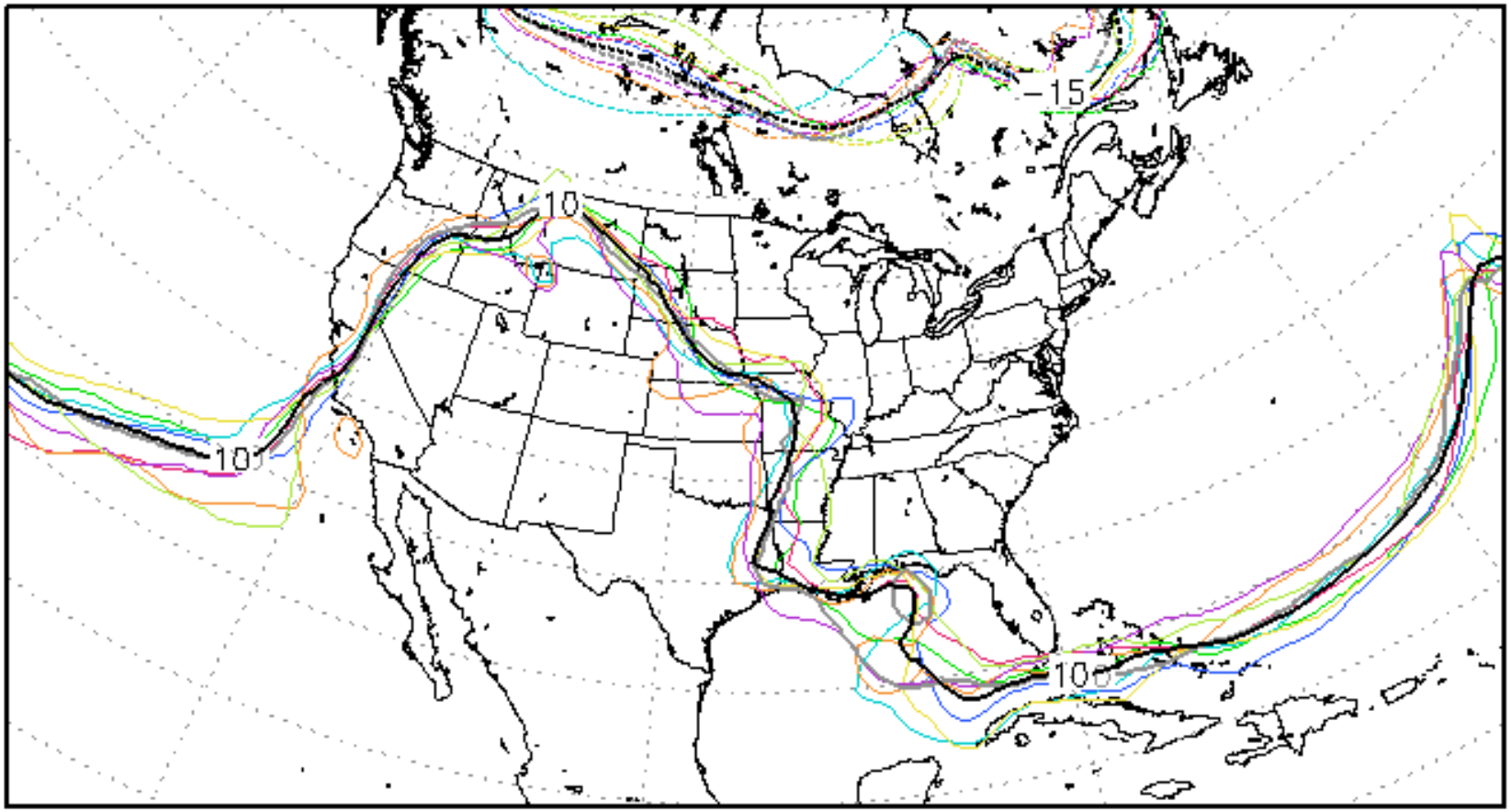


Minneapolis,
Mar 22 – Mar 28
2005

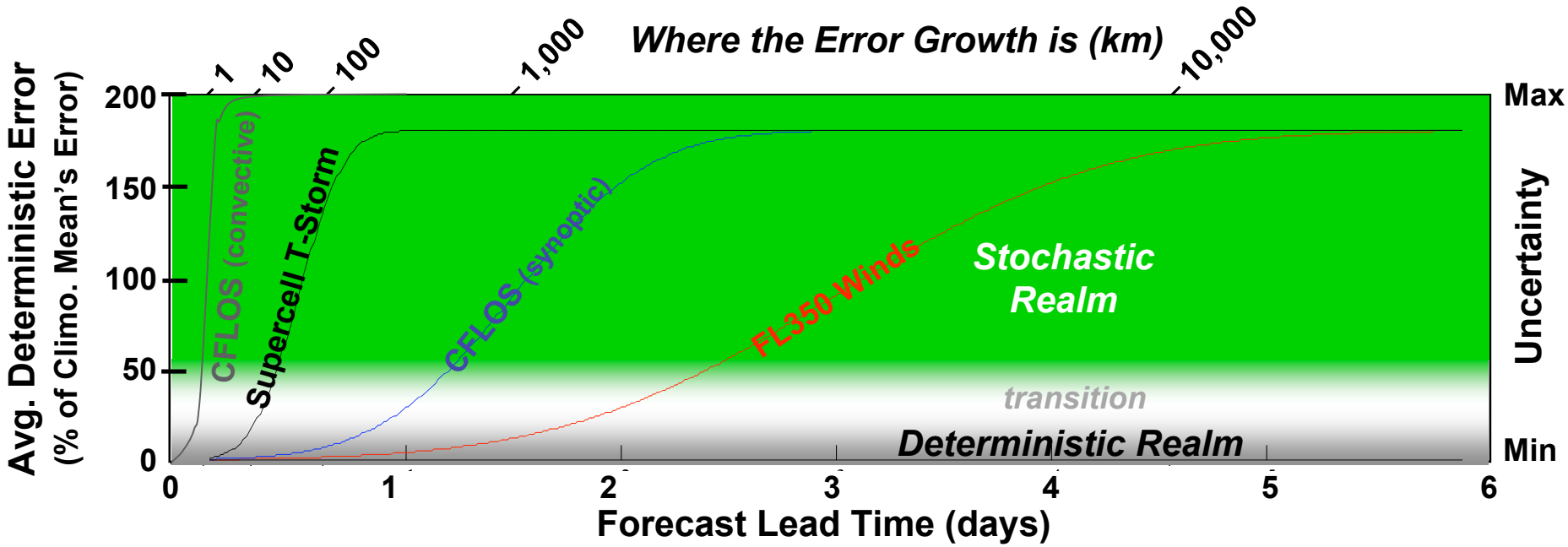


63
44

ens run for 12Z 22Mar2004 valid 00Z25MAR2004 −15 and 10 temp. contours at 850−hPa, 1x1 degree resolution

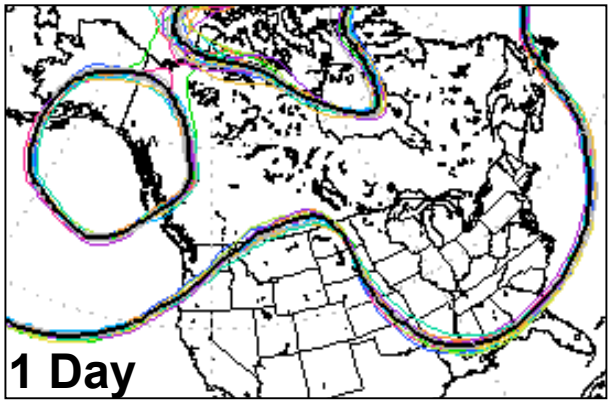


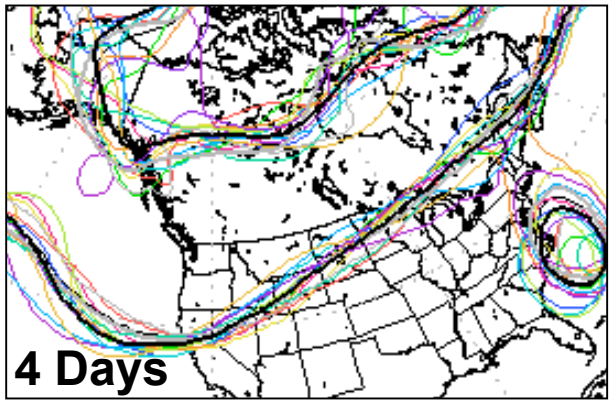
Scale Dependent Error Growth

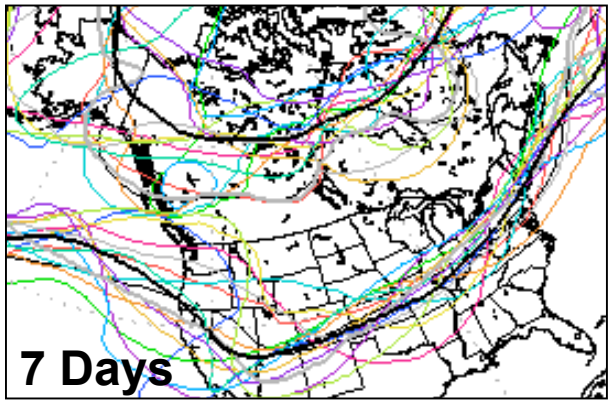
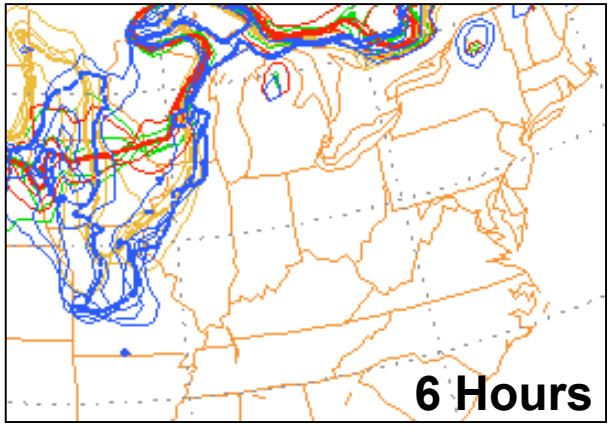
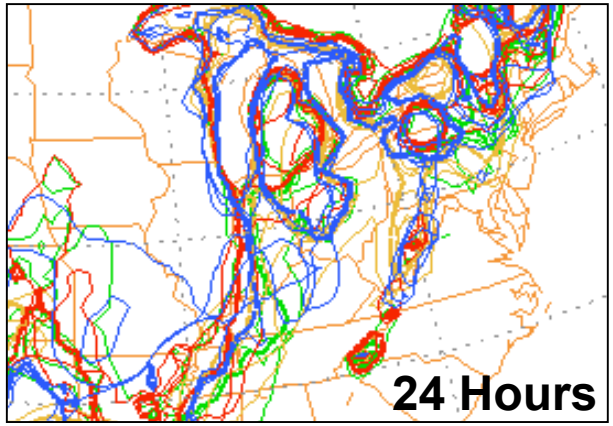
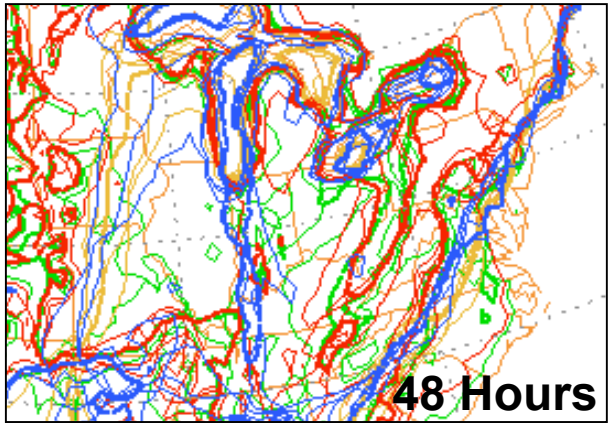


Scale Dependent Error Growth

NCEP Global, 15-Member Ensemble 00Z, 19 Nov 2006
500Hpa GPH, 522 and 564 Contour Spaghetti







NCEP CONUS, 21-Member Ensemble 09Z, 19 Nov 2006
Surface Temp, 0°C Isotherm Spaghetti